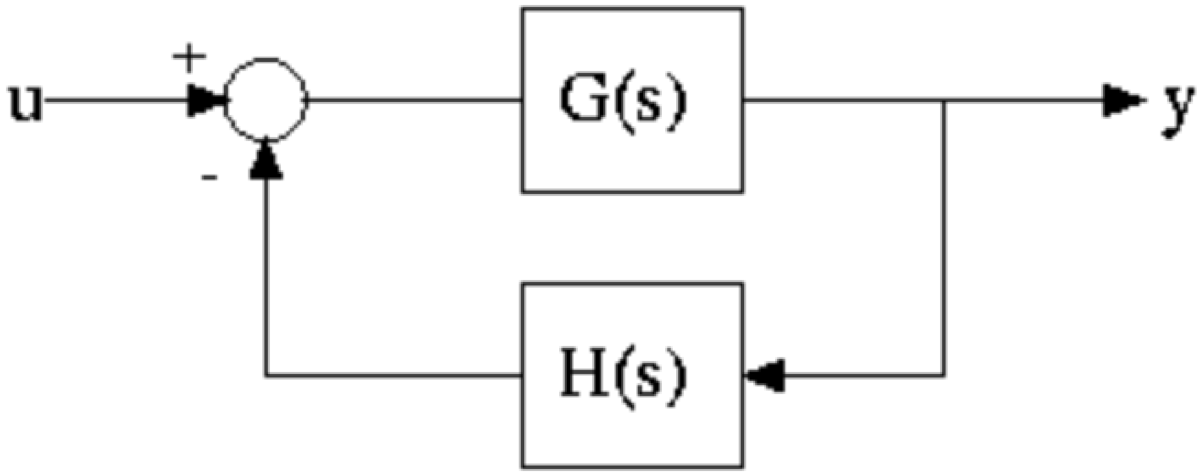
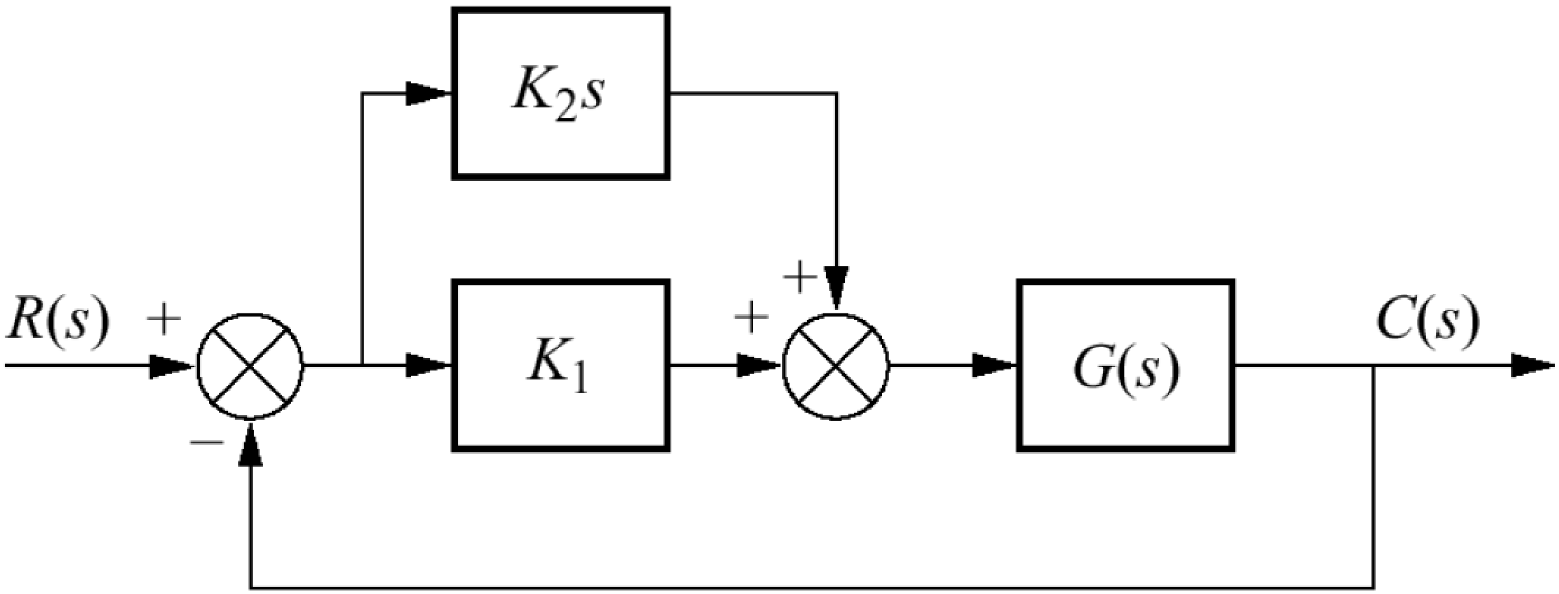
System Control

Design via root locus



PD controller implementation



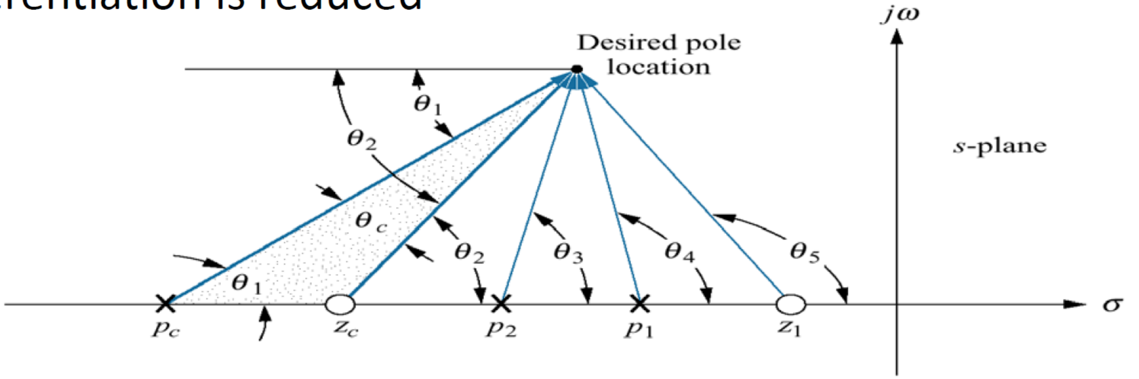
$$G_c(s) = K_2 s + K_1 = K_2\left(s + \frac{K_1}{K_2}\right)$$

K_2 is chosen to contribute to the required loop-gain value. And K_1/K_2 is chosen to equal the negative of the compensator zero.

Geometry of lead compensation

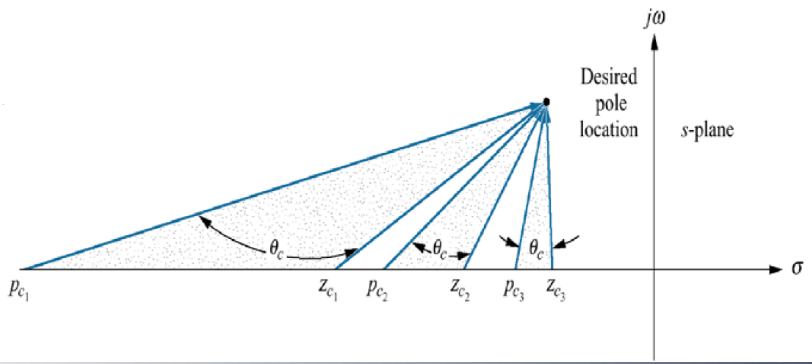
Advantages of a passive lead network over an active PD controller:

1) no need for additional power supply
2) noise due to differentiation is reduced



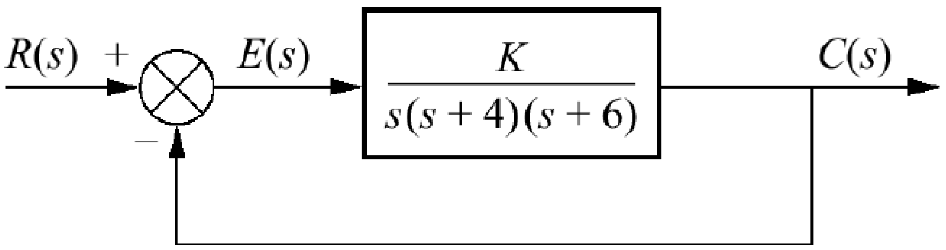
$$\theta_2 - \theta_1 - \theta_3 - \theta_4 + \theta_5 = (2k+1)180^\circ$$

Three of the infinite possible lead compensator solutions



Lead compensator design, Example 9.4

Problem: Design 3 lead compensators for the system in figure that will reduce the settling time by a factor of 2 while maintaining 30% overshoot.



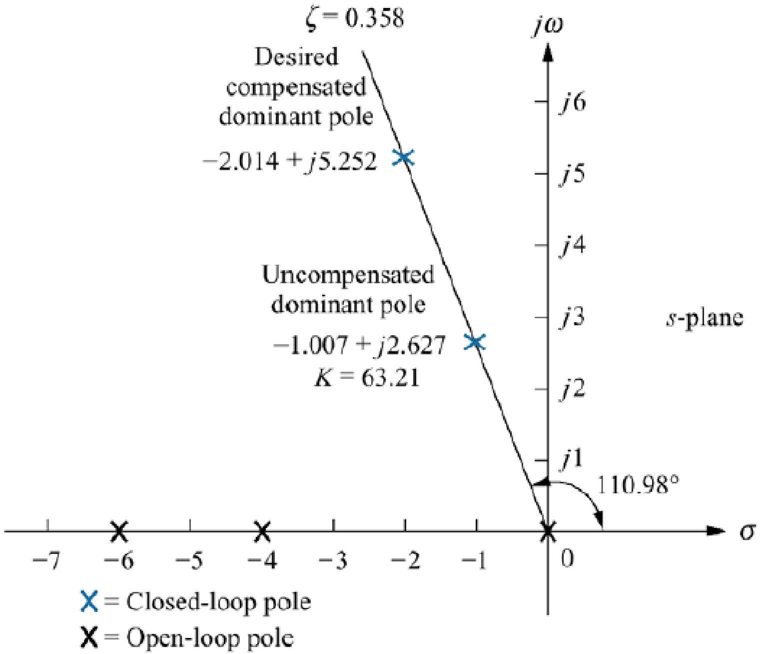
Solution: The uncompensated settling time is $T_s = \frac{4}{\xi\omega_n} = \frac{4}{1.007} = 3.972$

To find the design point, new settling time is $T_s = \frac{3.972}{2} = 1.986$

From which the real part of the desired pole location is $\sigma = \frac{4}{T_s} = \frac{4}{1.986} = 2.014$

And the imaginary part is

$$\omega_d = 2.014\tan(110.98^\circ) = 5.252$$

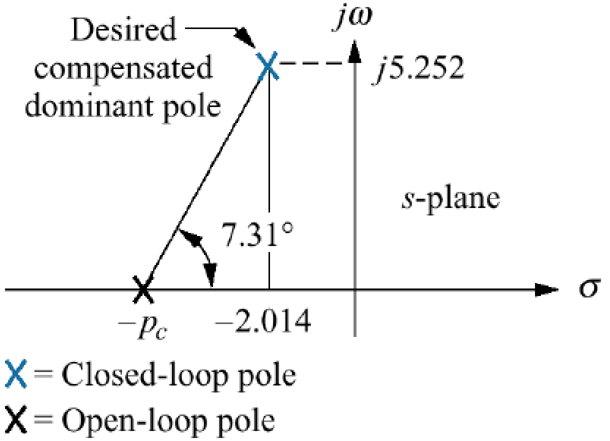


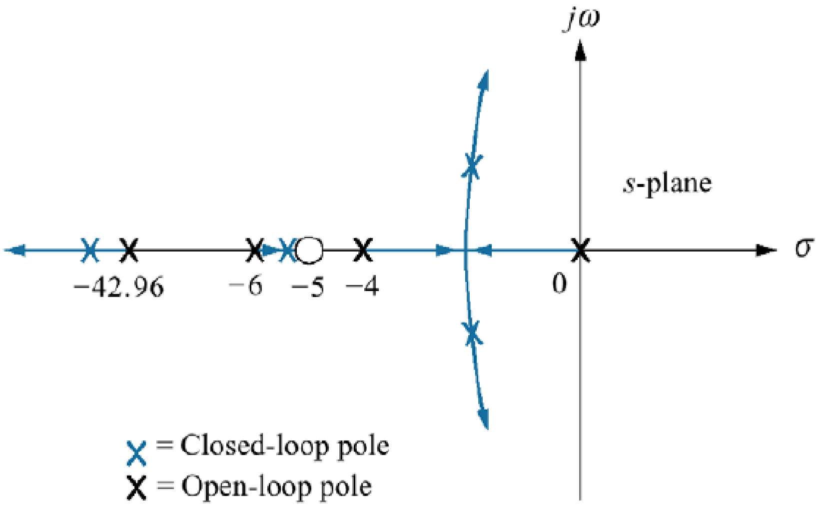
S-plan picture used to calculate the location of the compensator pole for Example 9.4

Arbitrarily assume a compensator zero at -5 on the real axis as possible solution. Then we find the compensator pole location as shown in figure.

Note sum of angles of compensator zero and all uncompensated poles and zeros is -172,69 so the angular contribution of the compensator pole is -7.31.

$$\frac{5.252}{p_c - 2.014} = \tan 7.31^\circ \quad \text{and } p_c = 42.96$$





Note: This figure is not drawn to scale.

Comparison of lead compensation designs for Example 9.4

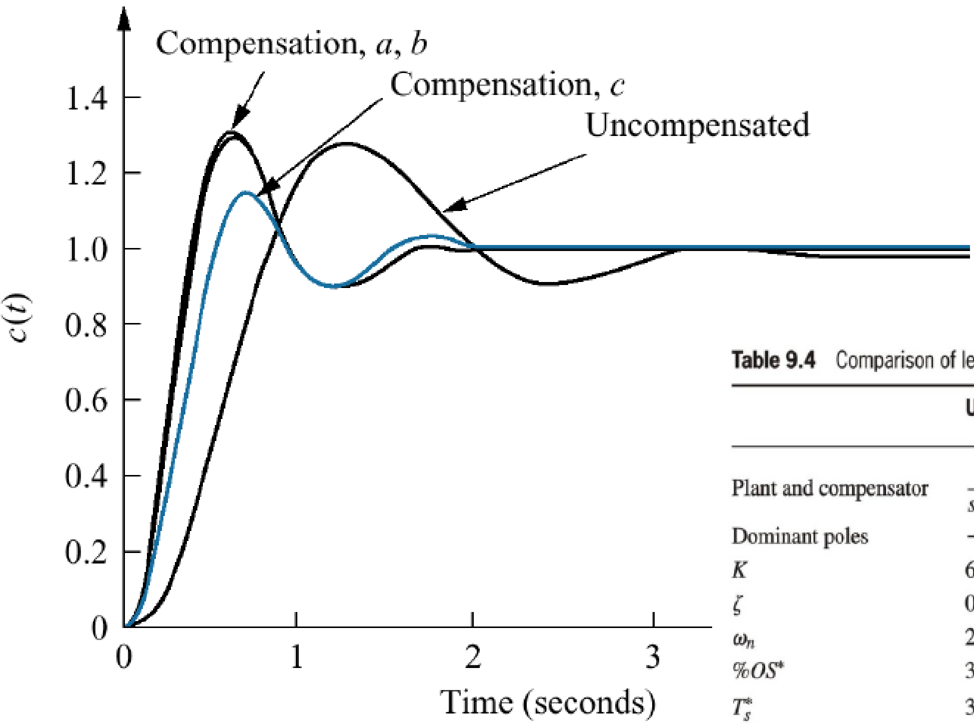


Table 9.4 Comparison of lead compensation designs for Example 9.4

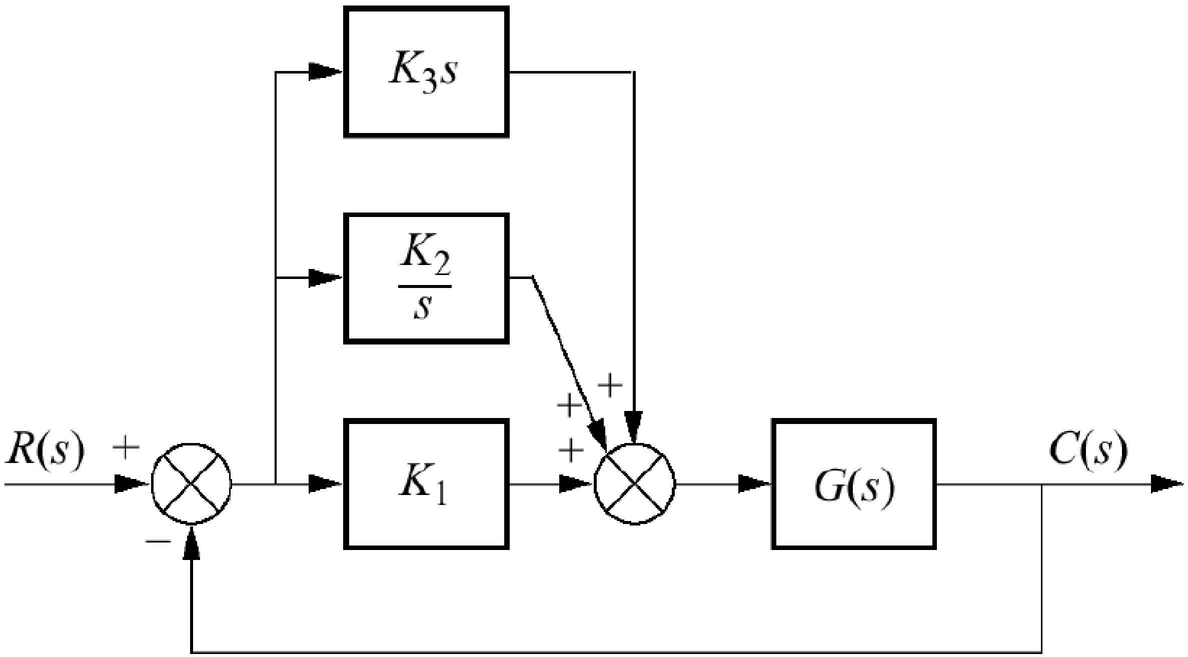
	Uncompensated	Compensation a	Compensation b	Compensation c
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$	$\frac{K(s+5)}{s(s+4)(s+6)(s+42.96)}$	$\frac{K(s+4)}{s(s+4)(s+6)(s+20.09)}$	$\frac{K(s+2)}{s(s+4)(s+6)(s+8.971)}$
Dominant poles	$-1.007 \pm j2.627$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$
K	63.21	1423	698.1	345.6
ζ	0.358	0.358	0.358	0.358
ω_n	2.813	5.625	5.625	5.625
$\%OS^*$	30 (28)	30 (30.7)	30 (28.2)	30 (14.5)
T_s^*	3.972 (4)	1.986 (2)	1.986 (2)	1.986 (1.7)
T_p^*	1.196 (1.3)	0.598 (0.6)	0.598 (0.6)	0.598 (0.7)
K_v	2.634	6.9	5.791	3.21
$e(\infty)$	0.380	0.145	0.173	0.312
Other poles	-7.986	-43.8, -5.134	-22.06	-13.3, -1.642
Zero	None	-5	None	-2
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK	No pole-zero cancellation

* Simulation results are shown in parentheses.

Improving Steady-State Error and Transient Response

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 + K_2 + K_3 s^2}{s} = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s}$$

PID controller or using passive network it's called lag-lad compensator



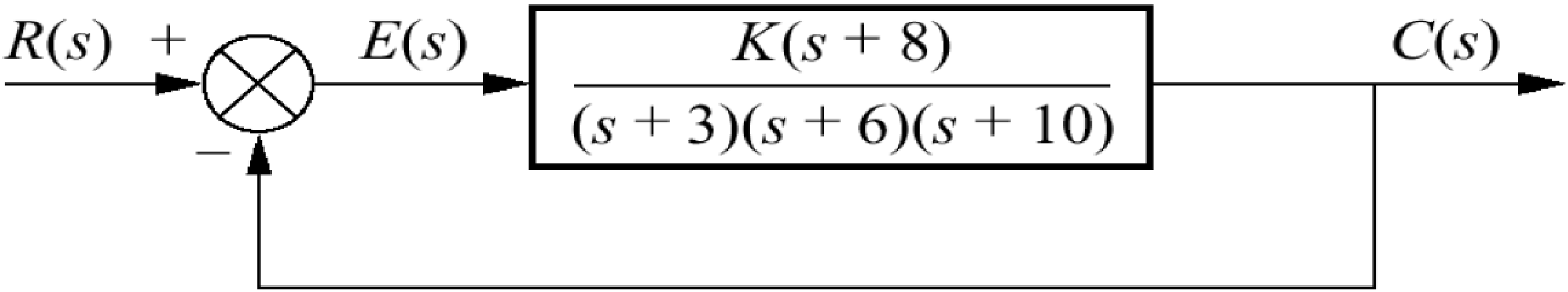
PID controller design

Design Steps:

- Evaluate the performance of the uncompensated system to determine how much improvement is required in transient response
- Design the PD controller to meet the transient response specifications. The design includes the zero location and the loop gain.
- Simulate the system to be sure all requirements have been met.
- Redesign if the simulation shows that requirements have not been met.
- Design the PI controller to yield the required steady-sate error.
- Determine the gains, K1, K2, and K3 shown in previous figure.
- Simulate the system to be sure all requirements have been met.
- Redesign if simulation shows that requirements have not been met.

PID controller design Example 9.5

Problem: Using the system in the Figure, Design a PID controller so that the system can operate with a peak time that is 2/3 that of the uncompensated system at 20% overshoot and with zero steady-state error for a step input



Solution: The uncompensated system operating at 20% overshoot has dominant poles at -5.415+j10.57 with gain 121.5, and a third pole at -8.169. The complete performance is shown in next table.

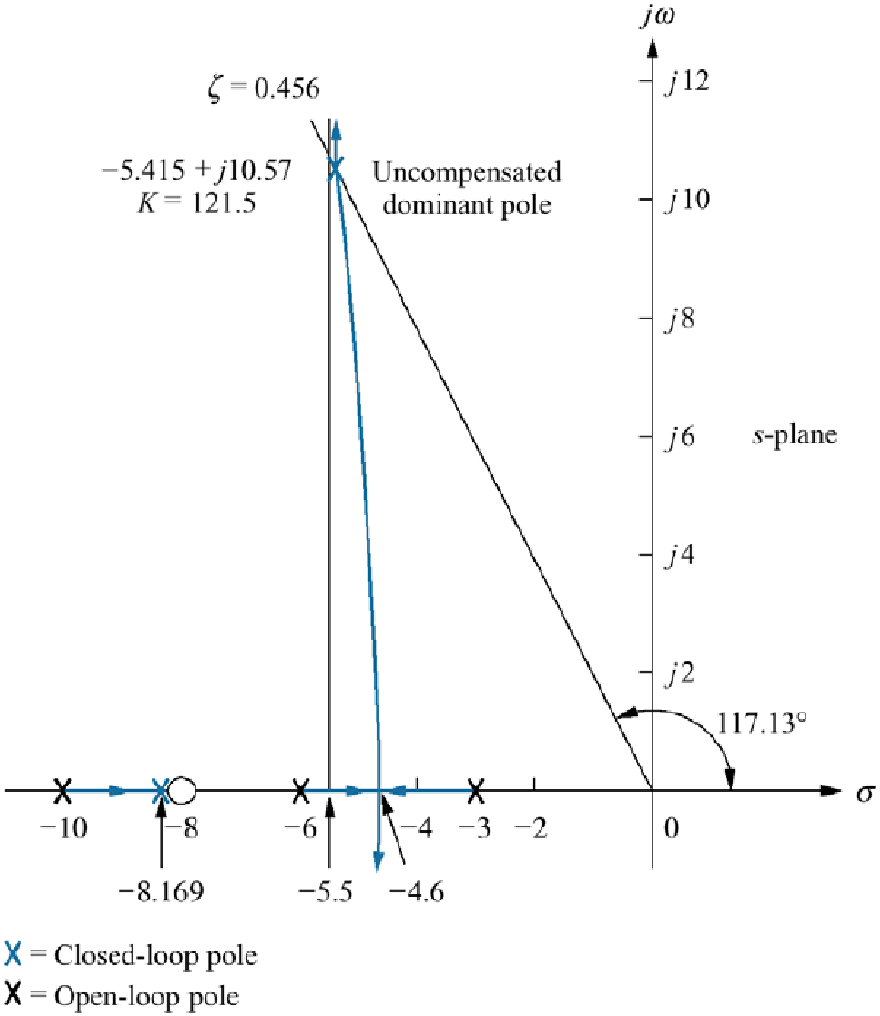
Root locus for the uncompensated system of Example 9.5

To compensate the system to reduce the peak time to 2/3 of original, we must find the compensated system dominant pole location. The imaginary part of the dominant pole is

$$\omega_d = \frac{\pi}{T_p} = \frac{\pi}{(2/3)(0.297)} = 15.87$$

Thus the real part is

$$\sigma = \frac{\omega_d}{\tan 117.13^\circ} = -8.13$$

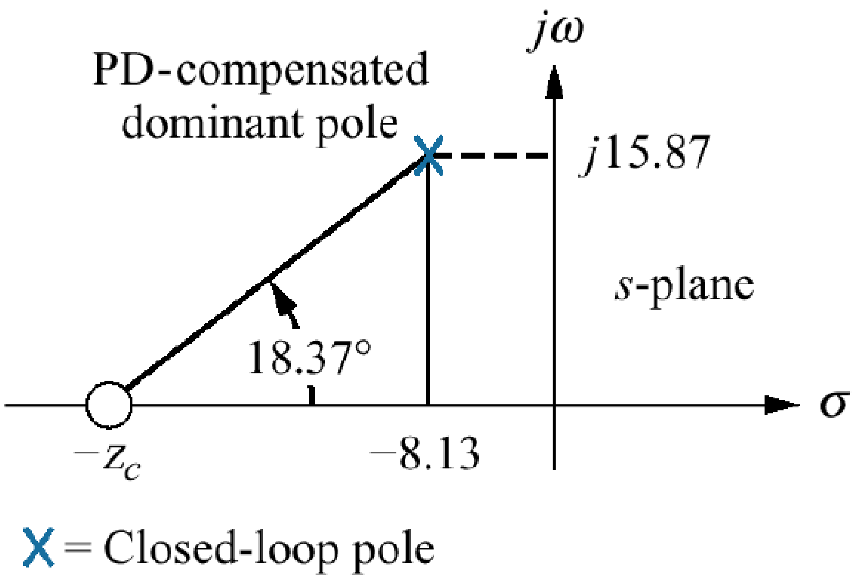


Predicted characteristics of uncompensated, PD- and PID- compensated systems of Example 9.5

	Uncompensated	**PD-compensated**	**PID-compensated**
Plant and compensator	$\dfrac{K(s+8)}{(s+3)(s+6)(s+10)}$	$\dfrac{K(s+8)(s+55.92)}{(s+3)(s+6)(s+10)}$	$\dfrac{K(s+8)(s+55.92)(s+0.5)}{(s+3)(s+6)(s+10)s}$
Dominant poles	$-5.415 \pm j10.57$	$-8.13 \pm j15.87$	$-7.516 \pm j14.67$
K	121.5	5.34	4.6
ζ	0.456	0.456	0.456
ω_n	11.88	17.83	16.49
$\%OS$	20	20	20
T_s	0.739	0.492	0.532
T_p	0.297	0.198	0.214
K_p	5.4	13.27	∞
$e(\infty)$	0.156	0.070	0
Other poles	-8.169	-8.079	$-8.099,\ -0.468$
Zeros	-8	$-8,\ -55.92$	$-8,\ -55.92,\ -0.5$
Comments	Second-order approx. OK	Second-order approx. OK	Zero at -55.92 and -0.5 not canceled

Calculating the PD compensator zero for Example 9.5

To design the compensator, we find the sum of angles from the uncompensated system's poles and zeros to the desired compensated dominant pole to be -198.37. Thus the contribution required from the compensator zero is 198.37-180=18.37. Then we calculate the location of the zero as:

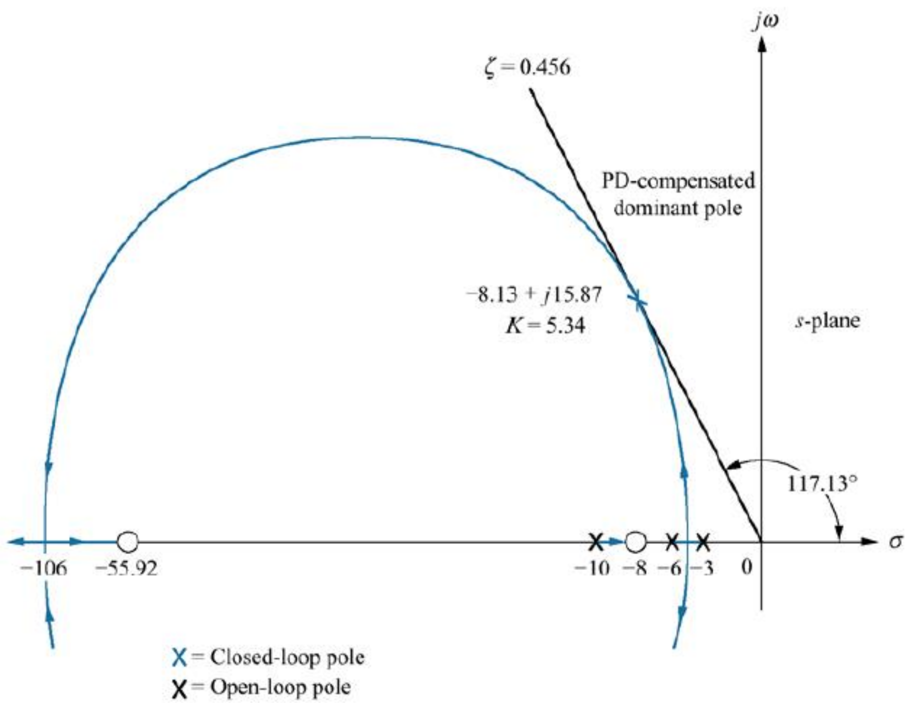


Note: This figure is not drawn to scale.

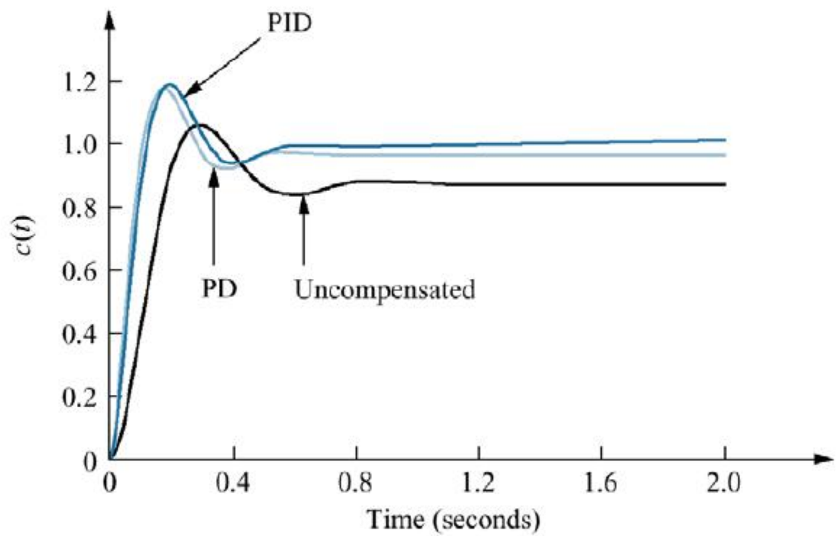
Thus the PD controller is $G_{PD}(s) = (s+55.92)$

The complete root locus sketch is shown in next slide. Using program the gain at the design point is 5.34

Root locus for PD-compensated system of Example 9.5



Note: This figure is not drawn to scale.

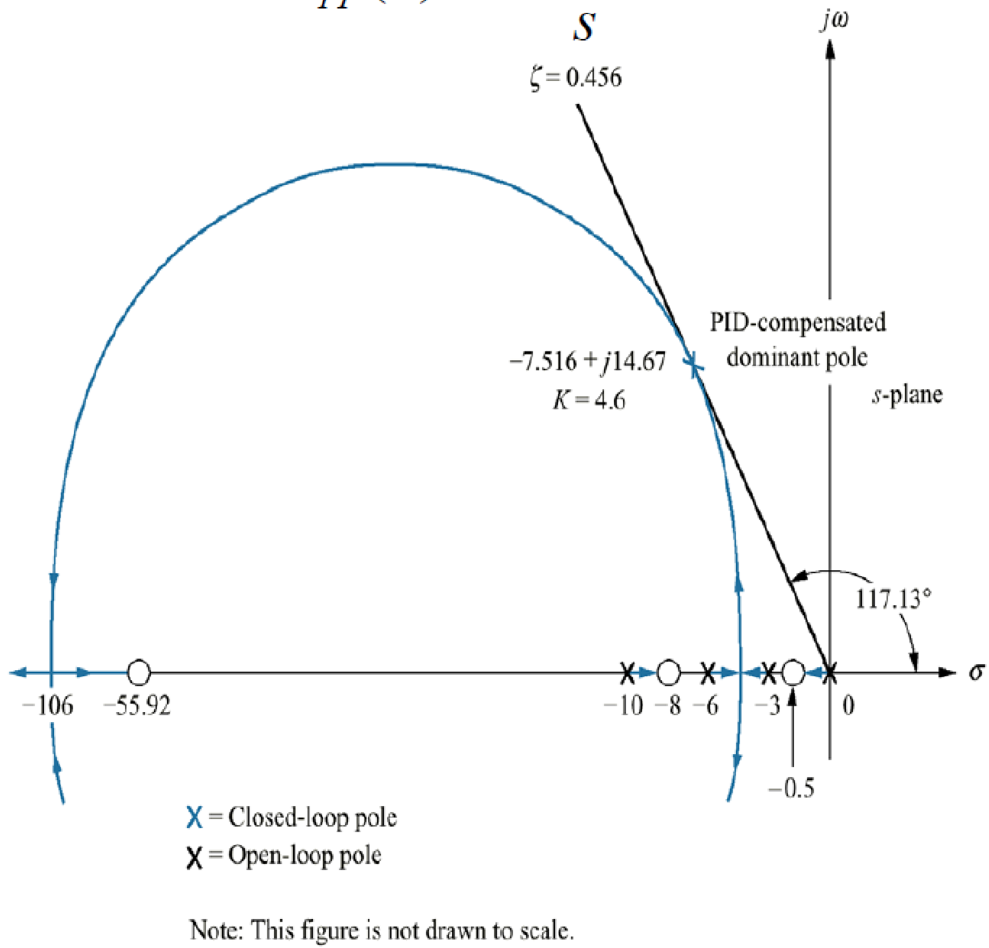


Root locus for PID-compensated system of Example 9.5

Choosing the ideal integral compensator to be $G_{PI}(s) = \dfrac{s+0.5}{s}$

And sketching the root locus for the PID-compensated system as shown. Searching the 0.456 damping ratio line, we find the dominant poles at -7.516+j14.67

The characteristics of the PID compensated system are shown in table.



Predicted characteristics of uncompensated, PD-, and PID- compensated systems of Example 9.5

	Uncompensated	PD-compensated	PID-compensated
Plant and compensator	$\dfrac{K(s+8)}{(s+3)(s+6)(s+10)}$	$\dfrac{K(s+8)(s+55.92)}{(s+3)(s+6)(s+10)}$	$\dfrac{K(s+8)(s+55.92)(s+0.5)}{(s+3)(s+6)(s+10)s}$
Dominant poles	$-5.415 \pm j10.57$	$-8.13 \pm j15.87$	$-7.516 \pm j14.67$
K	121.5	5.34	4.6
ζ	0.456	0.456	0.456
ω_n	11.88	17.83	16.49
$\%OS$	20	20	20
T_s	0.739	0.492	0.532
T_p	0.297	0.198	0.214
K_p	5.4	13.27	∞
$e(\infty)$	0.156	0.070	0
Other poles	-8.169	-8.079	$-8.099,\ -0.468$
Zeros	-8	$-8,\ -55.92$	$-8,\ -55.92,\ -0.5$
Comments	Second-order approx. OK	Second-order approx. OK	Zero at -55.92 and -0.5 not canceled

Improving Steady-State Error and Transient Response

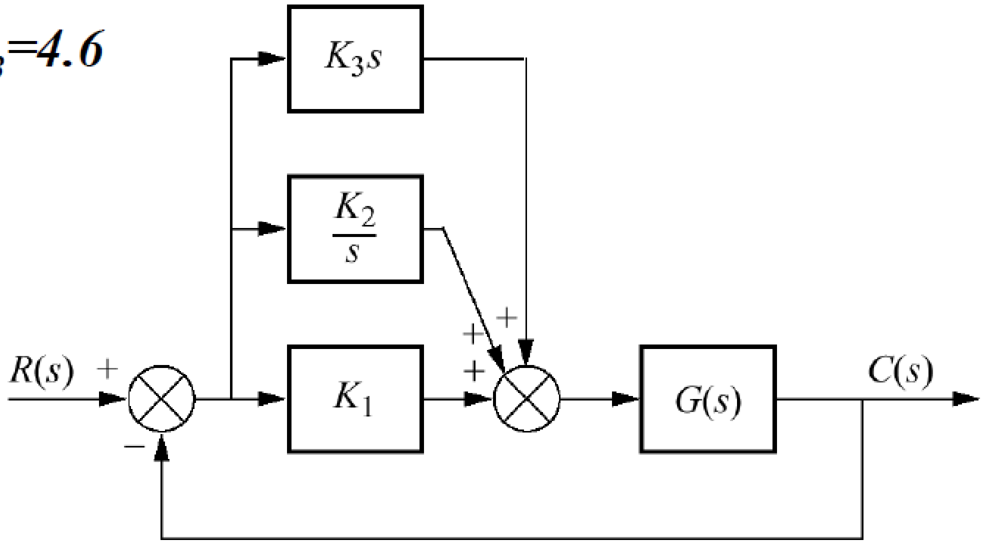
Finally to implement the compensator and find the K's, using the PD and PI compensators

$$G_{PID}(s) = \frac{K(s+55.92)(s+0.5)}{s} = \frac{4.6(s^2+56.42s+27.96)}{s}$$

and compare to

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 + K_2 + K_3 s^2}{s} = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s}$$

we find K_1= 259.5, K_2=128.6, and K_3=4.6



Lag-Lead compensator design

1. Evaluate the performance of the uncompensated system to determine how much improvement in transient response is required.
2. Design the lead compensator to meet the transient response specifications. The design includes the zero location, pole location, and the loop gain.
3. Simulate the system to be sure all requirements have been met.
4. Redesign if the simulation shows that requirements have not been met.
5. Evaluate the steady-state error performance for the lead-compensated system to determine how much more improvement in steady-state error is required.
6. Design the lag compensator to yield the required steady-state error.
7. Simulate the system to be sure all requirements have been met.
8. Redesign if the simulation shows that requirements have not been met.

Example 9.6

PROBLEM: Design a lag-lead compensator for the system of Figure 9.37 so that the system will operate with 20% overshoot and a twofold reduction in settling time. Further, the compensated system will exhibit a tenfold improvement in steady-state error for a ramp input.

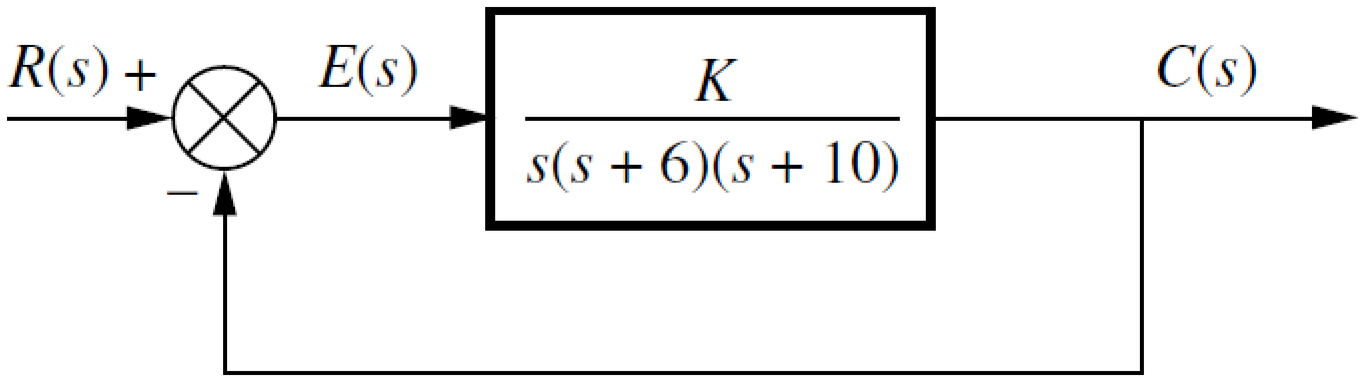


FIGURE 9.37 Uncompensated system for Example 9.6

Example 9.6

Step 1 First we evaluate the performance of the uncompensated system. Searching along the 20% overshoot line ($\zeta = 0.456$) in Figure 9.38, we find the dominant poles at $-1.794 \pm j3.501$, with a gain of 192.1. The performance of the uncompensated system is summarized in Table 9.6.

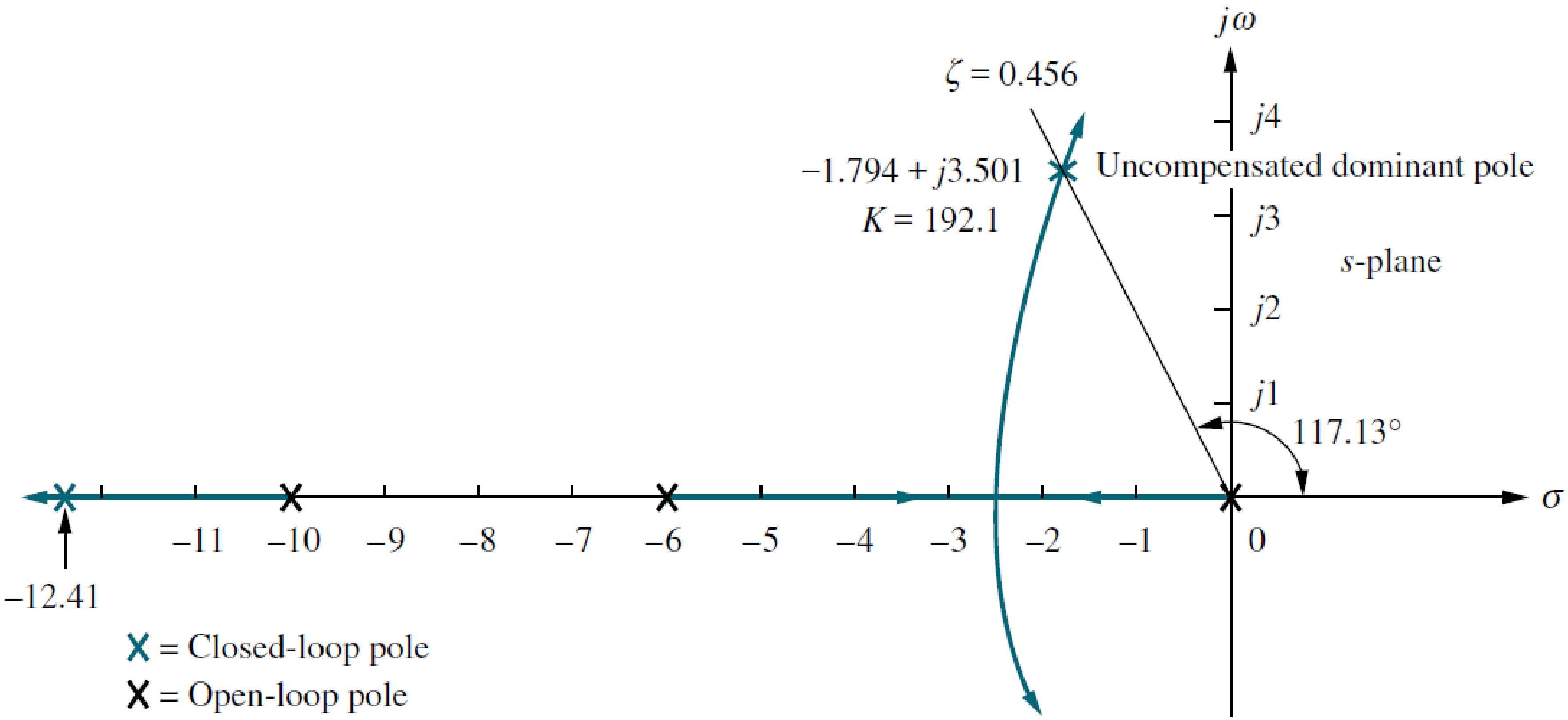
Step 2 Next we begin the lead compensator design by selecting the location of the compensated system's dominant poles. In order to realize a twofold reduction in settling time, the real part of the dominant pole must be increased by a factor of 2, since the settling time is inversely proportional to the real part. Thus,

$$-\zeta\omega_n = -2(1.794) = -3.588 \tag{9.29}$$

The imaginary part of the design point is

$$\omega_d = \zeta\omega_n \tan 117.13^\circ = 3.588 \tan 117.13^\circ = 7.003 \tag{9.30}$$

Example 9.6



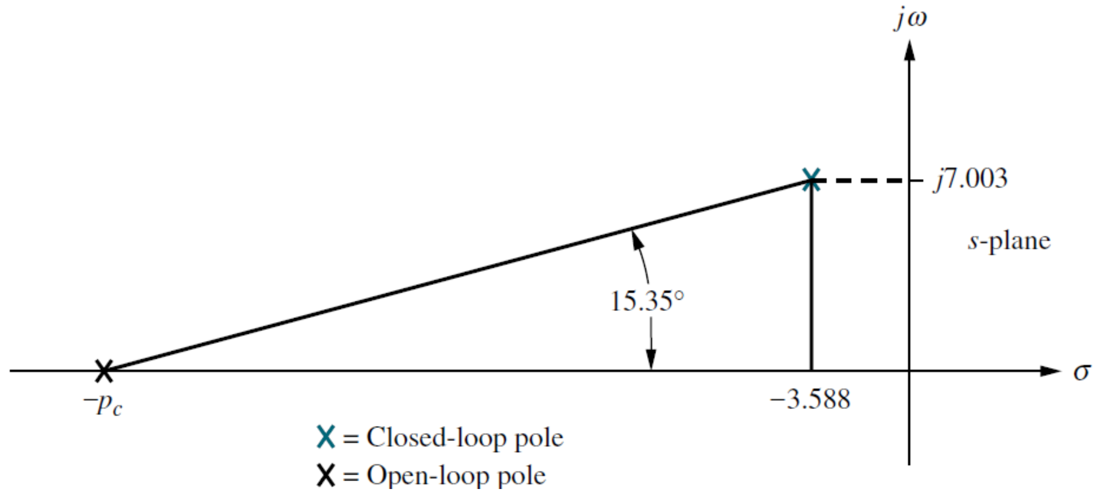
jω
ζ = 0.456
j4
−1.794 + j3.501
K = 192.1
Uncompensated dominant pole
j3
s-plane
j2
j1
117.13°
σ
−11
−10
−9
−8
−7
−6
−5
−4
−3
−2
−1
0
−12.41
= Closed-loop pole
= Open-loop pole

Example 9.6

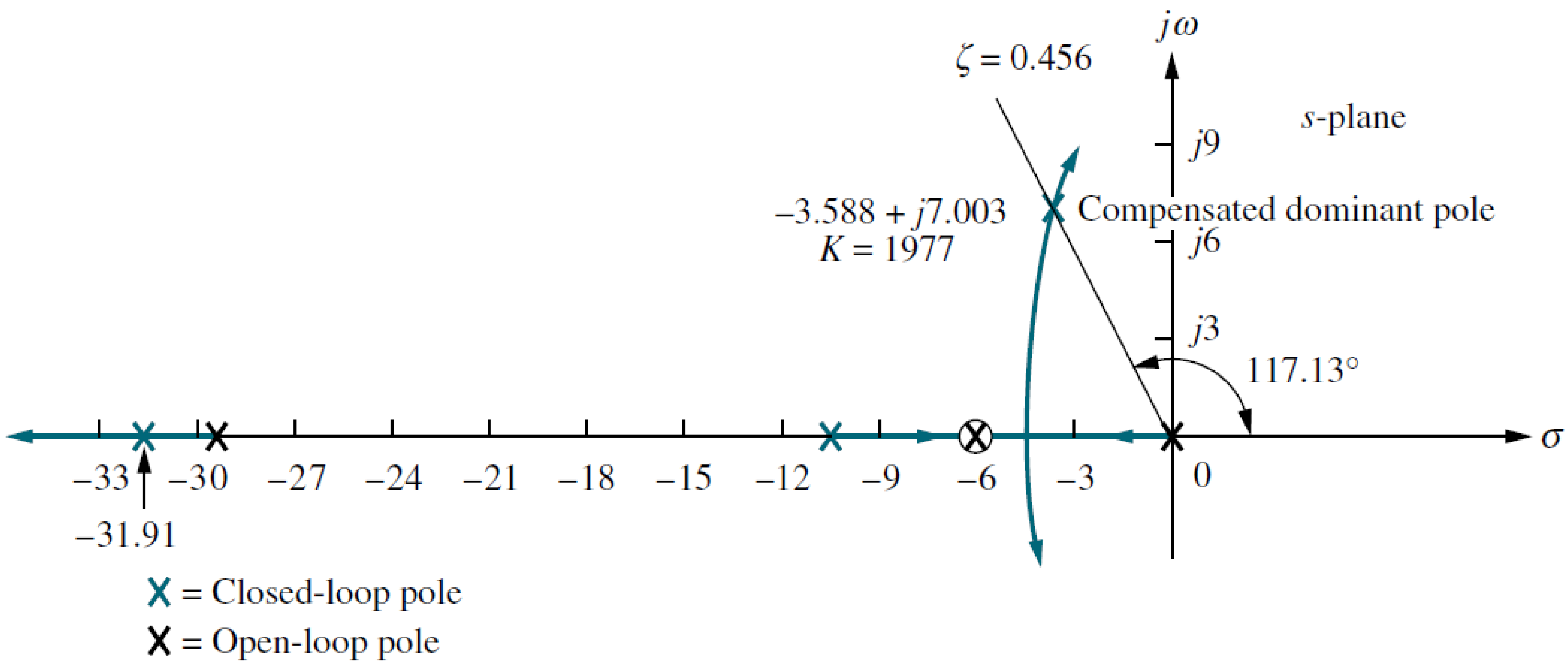
We complete the design by finding the location of the compensator pole. Using the root locus program, sum the angles to the design point from the uncompensated system's poles and zeros and the compensator zero and get -164.65°. The difference between 180° and this quantity is the angular contribution required from the compensator pole, or -15.35°. Using the geometry shown in Figure 9.39,

$$\frac{7.003}{p_c - 3.588} = \tan 15.35^\circ \tag{9.31}$$

from which the location of the compensator pole, p_c, is found to be -29.1.



Example 9.6



Lead compensation

$$G(s) = \frac{192.1}{s(s+6)(s+10)} \longrightarrow G_{LC}(s) = \frac{1977}{s(s+10)(s+29.1)}$$

Example 9.6

$$G(s) = \frac{192.1}{s(s+6)(s+10)} \tag{9.32}$$

the static error constant, K_v, which is inversely proportional to the steady-state error, is 3.201. Since the open-loop transfer function of the lead-compensated system is

$$G_{\mathrm{LC}}(s) = \frac{1977}{s(s+10)(s+29.1)} \tag{9.33}$$

the static error constant, K_v, which is inversely proportional to the steady-state error, is 6.794. Thus, the addition of lead compensation has improved the steady-state error by a factor of 2.122. Since the requirements of the problem specified a tenfold improvement, the lag compensator must be designed to improve the steady-state error by a factor of 4.713 $(10/2.122 = 4.713)$ over the lead-compensated system.

Example 9.6

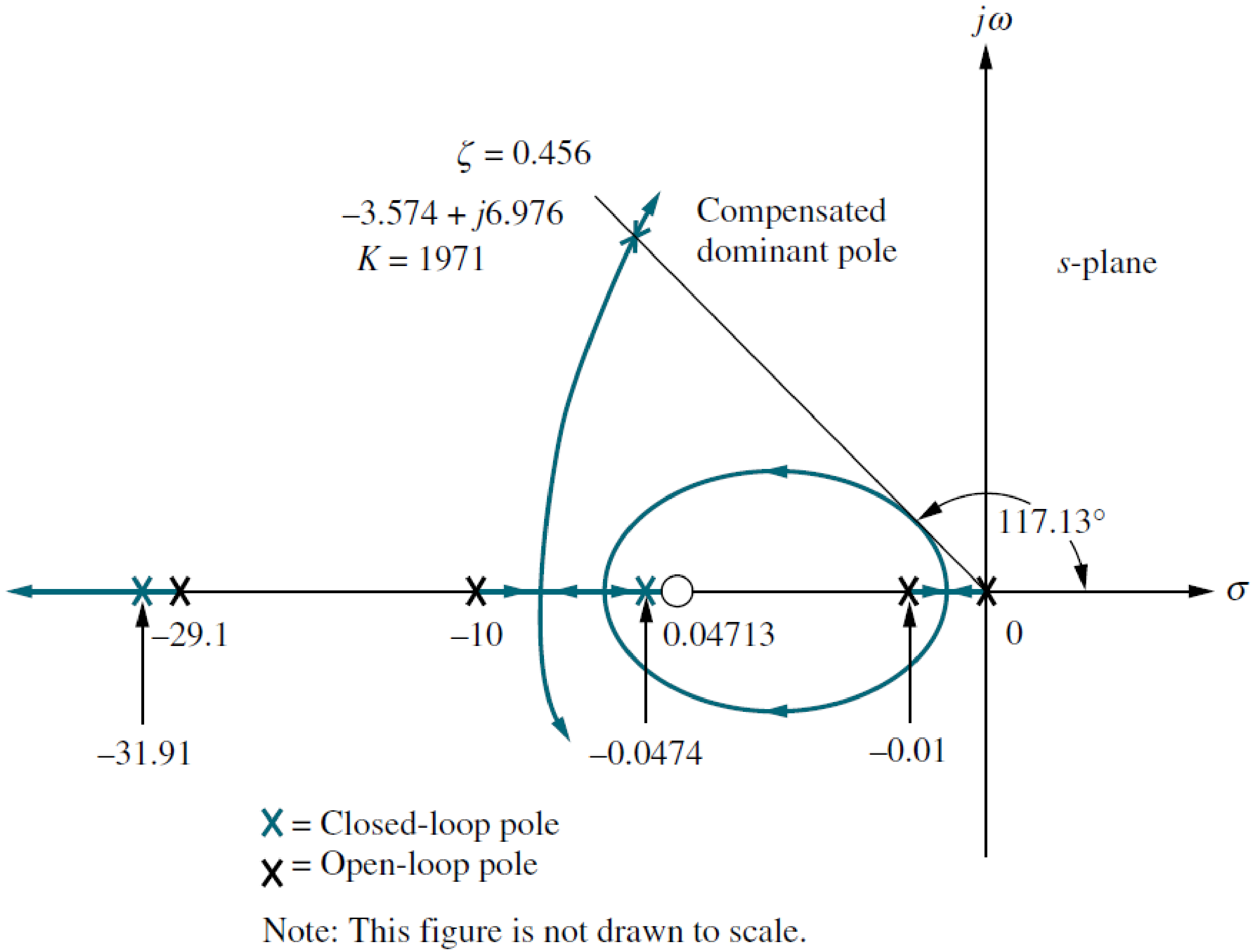
Step 6 We arbitrarily choose the lag compensator pole at 0.01, which then places the lag compensator zero at 0.04713, yielding

$$G_{\mathrm{lag}}(s) = \frac{(s+0.04713)}{(s+0.01)} \tag{9.34}$$

as the lag compensator. The lag-lead-compensated system's open-loop transfer function is

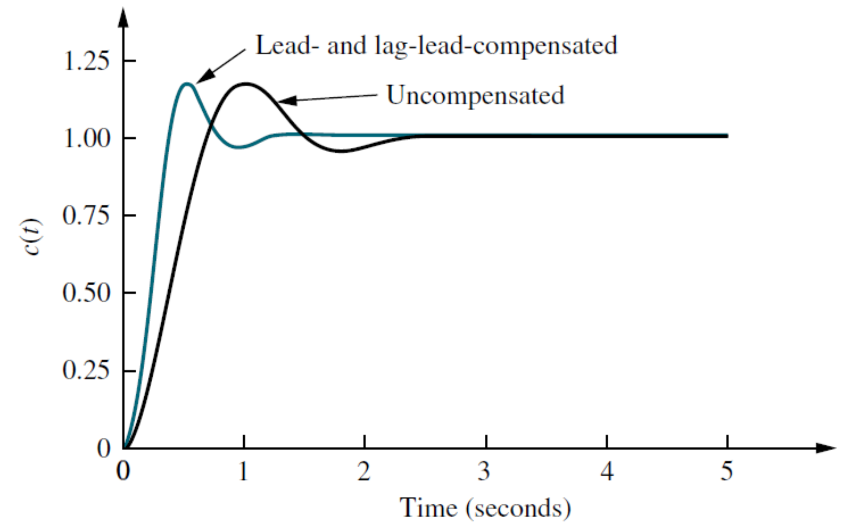
$$G_{\mathrm{LLC}}(s) = \frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)} \tag{9.35}$$

Example 9.6

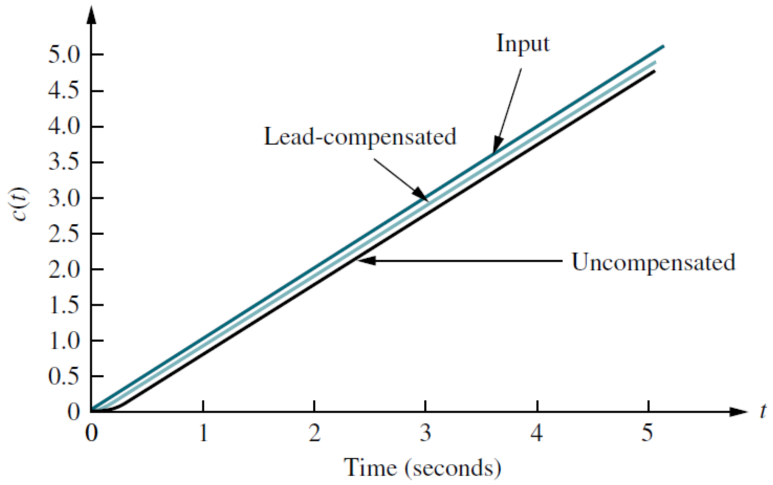


jω
ζ = 0.456
–3.574 + j6.976
K = 1971
Compensated
dominant pole
s-plane
117.13°
σ
–29.1
–10
0.04713
0
–31.91
–0.0474
–0.01
= Closed-loop pole
= Open-loop pole
Note: This figure is not drawn to scale.

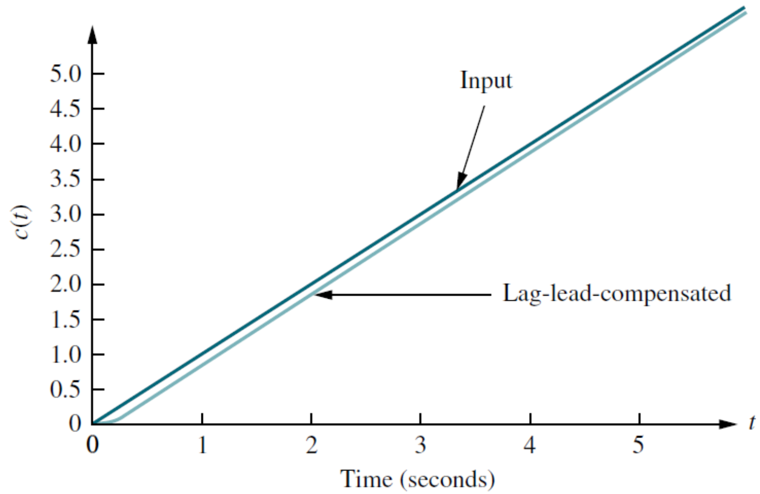
Example 9.6



Lead- and lag-lead-compensated
Uncompensated
1.25
1.00
0.75
0.50
0.25
0
c(t)
0 1 2 3 4 5
Time (seconds)

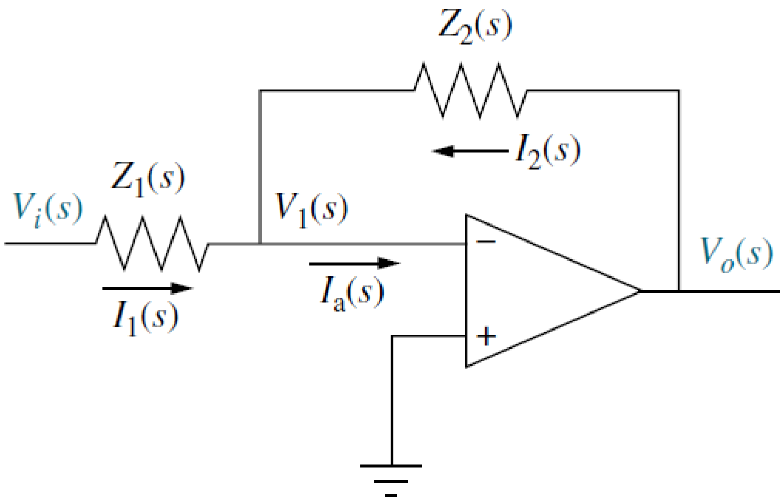


Input
Lead-compensated
Uncompensated
5.0 4.5 4.0 3.5 3.0 2.5 2.0 1.5 1.0 0.5 0
c(t)
0 1 2 3 4 5 t
Time (seconds)



Input
Lag-lead-compensated
5.0 4.5 4.0 3.5 3.0 2.5 2.0 1.5 1.0 0.5 0
c(t)
0 1 2 3 4 5 t
Time (seconds)

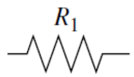
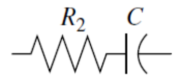
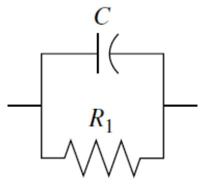
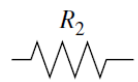
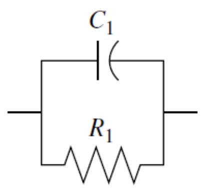
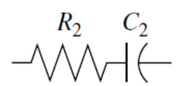
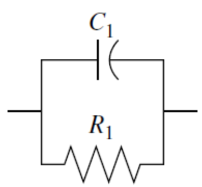
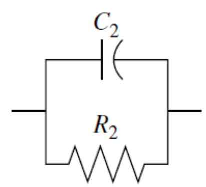
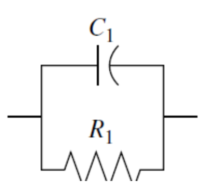
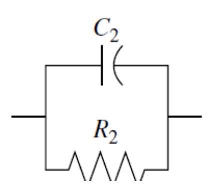
Active-Circuit Realization



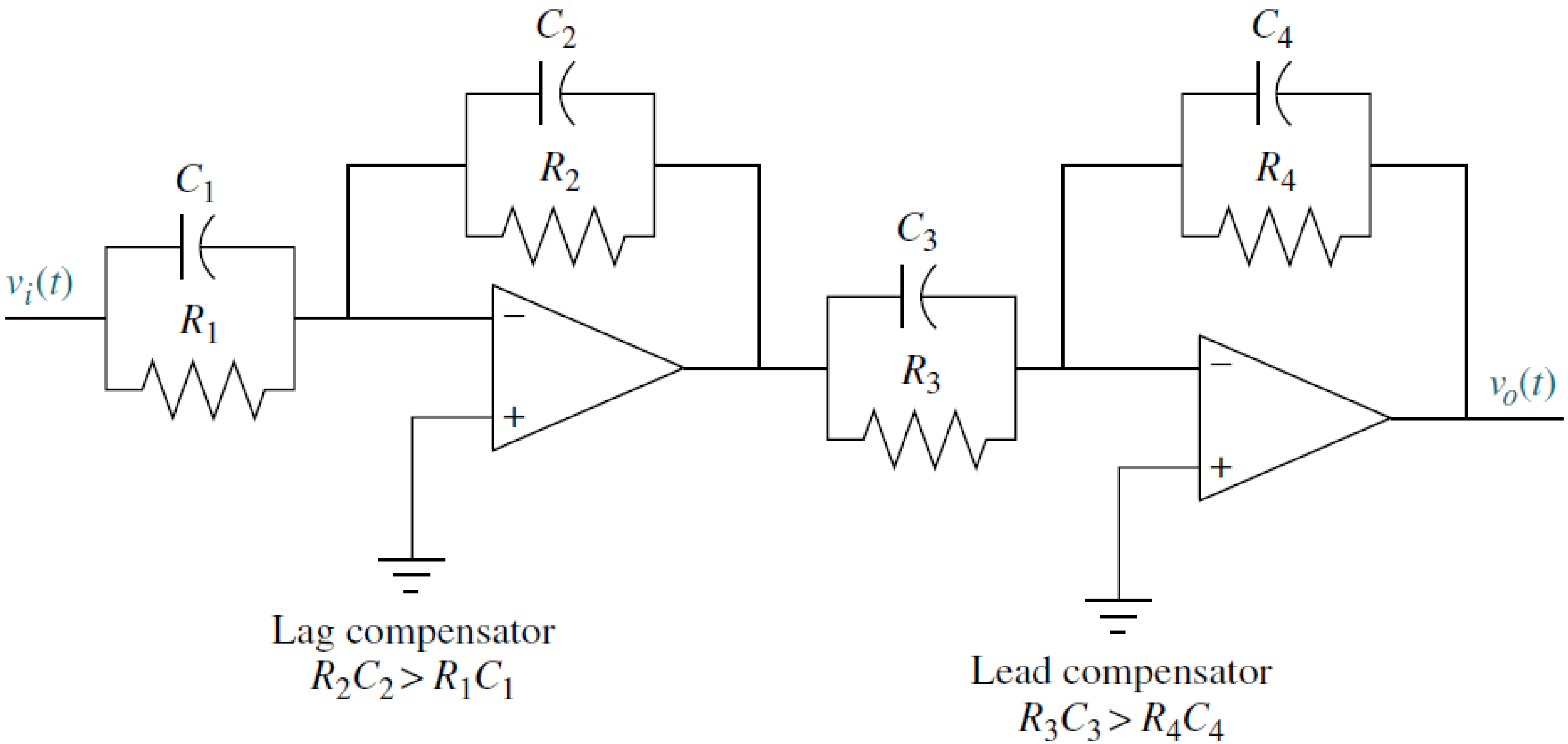
$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Gain	R_1	R_2	$-\frac{R_2}{R_1}$
Integration	R	C	$-\frac{\frac{1}{RC}}{s}$
Differentiation	C	R	$-RCs$

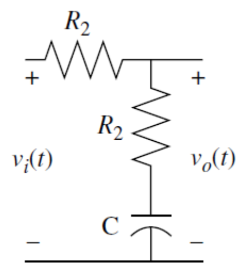
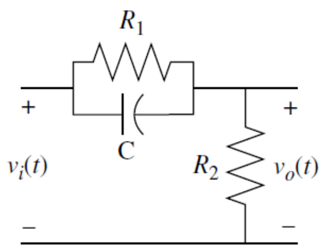
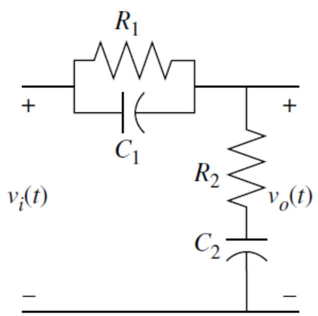
Active-Circuit Realization

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
PI controller			$-\dfrac{R_2}{R_1}\dfrac{\left(s+\dfrac{1}{R_2C}\right)}{s}$
PD controller			$-R_2C\left(s+\dfrac{1}{R_1C}\right)$
PID controller			$-\left[\left(\dfrac{R_2}{R_1}+\dfrac{C_1}{C_2}\right)+R_2C_1s+\dfrac{\dfrac{1}{R_1C_2}}{s}\right]$
Lag compensation			$-\dfrac{C_1}{C_2}\dfrac{\left(s+\dfrac{1}{R_1C_1}\right)}{\left(s+\dfrac{1}{R_2C_2}\right)}$ where $R_2C_2 > R_1C_1$
Lead compensation			$-\dfrac{C_1}{C_2}\dfrac{\left(s+\dfrac{1}{R_1C_1}\right)}{\left(s+\dfrac{1}{R_2C_2}\right)}$ where $R_1C_1 > R_2C_2$

Active-Circuit Realization



Passive-Circuit Realization

Function	Network	Transfer function, $\dfrac{V_o(s)}{V_i(s)}$
Lag compensation		$\dfrac{R_2}{R_1+R_2}\dfrac{s+\dfrac{1}{R_2C}}{s+\dfrac{1}{(R_1+R_2)C}}$
Lead compensation		$\dfrac{s+\dfrac{1}{R_1C}}{s+\dfrac{1}{R_1C}+\dfrac{1}{R_2C}}$
Lag-lead compensation		$\dfrac{\left(s+\dfrac{1}{R_1C_1}\right)\left(s+\dfrac{1}{R_2C_2}\right)}{s^2+\left(\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_2}+\dfrac{1}{R_2C_1}\right)s+\dfrac{1}{R_1R_2C_1C_2}}$