K-Digital Training

# ADAS센서원리 및 퓨전이해

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센서 종 류	설명	장점	단점	실제 영상 예시
카메라	2D 이미지 획 득	고해상도, 저렴 함	거리 측정 약 함, 빛에 민감	Lane detection 영상
레이더	전파 기반 거 리/속도 탐지	거리/속도 정 확, 날씨 강인	해상도 낮음	자동차 전방 추적
라이다	레이저 기반 3D 거리 정보	정밀한 3D 인 식	고가, 날씨 민 감	LiDAR Point Cloud 애니메이션
IMU/GPS	움직임 + 위치 정보	자율 위치 추정 에 필수	단독으론 누적 오차	GPS-IMU 통합 지도













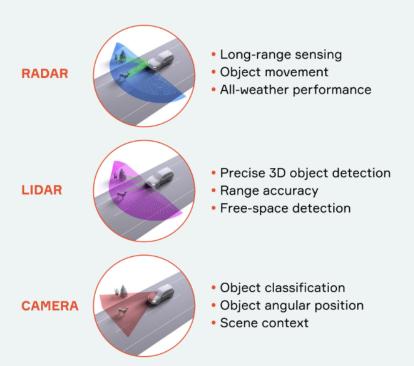
운전자가 수행 🔵 운전자가 조건부 수행 🌑 시스템이 수행





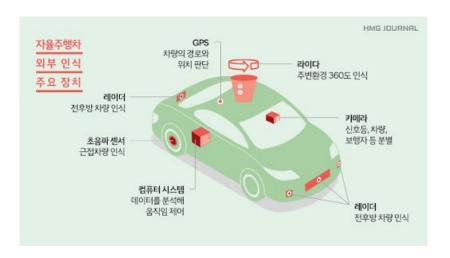
#### **Sensor Fusion**

Multiple sensing modalities required; sensor fusion brings them together



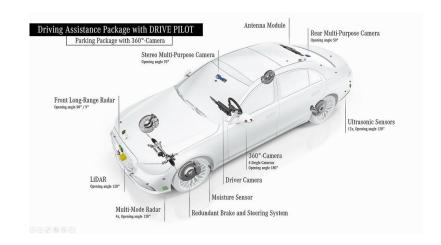
	RADAR	LIDAR	CAMERA	FUSION
Object detection	+	+	$\circ$	+
Pedestrian detection	_	0	+	+
Weather conditions	+	0	_	+
Lighting conditions	+	+	_	+
Dirt	+	0	_	+
Velocity	+	0	0	+
Distance - accuracy	+	+	$\circ$	+
Distance - range	+	0	0	+
Data density	_	0	+	+
Classification	_	0	+	+
Packaging	+	_	0	+

+ = Strength O = Capability — = Weakness





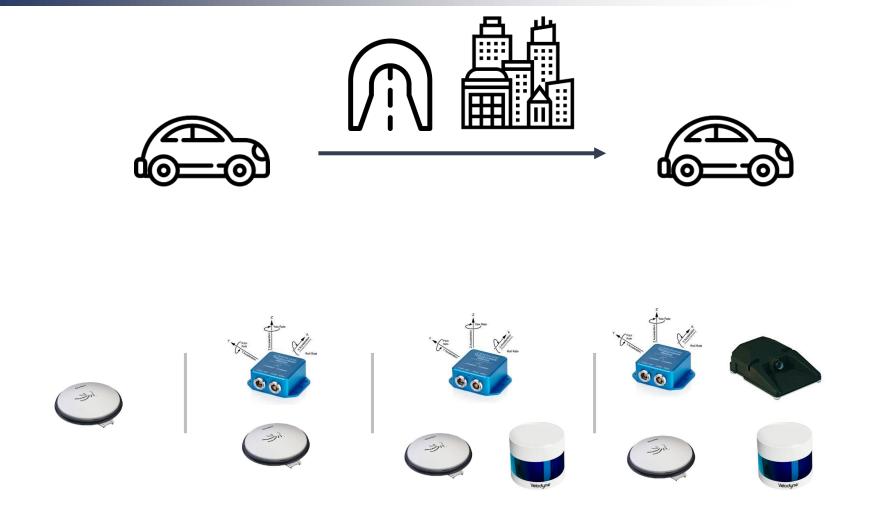




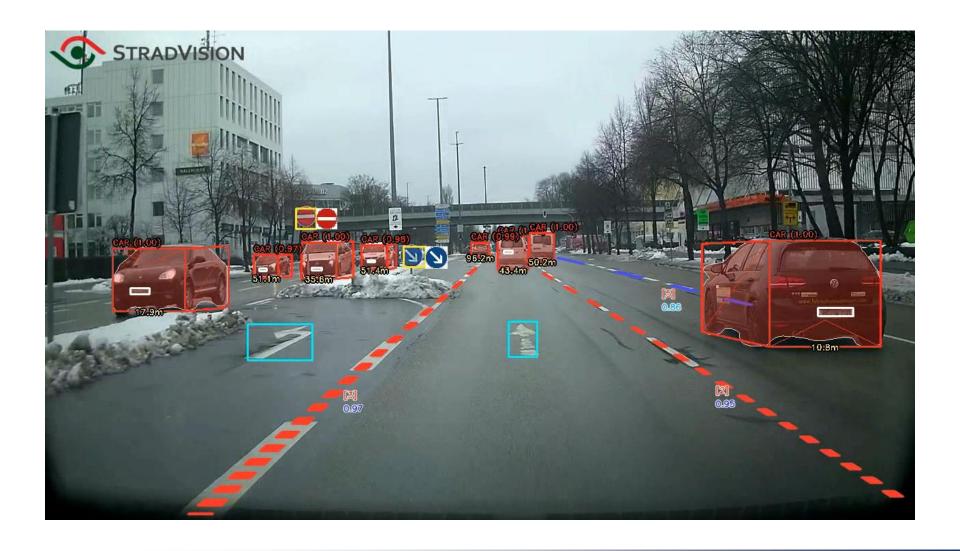




## Sensor fusion?

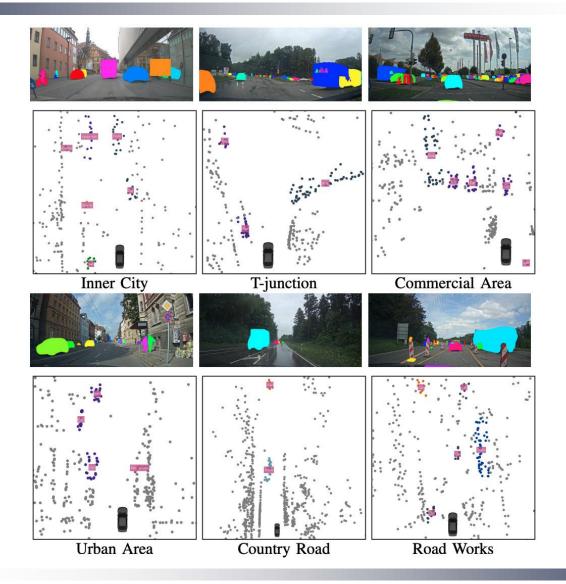


### Camera view

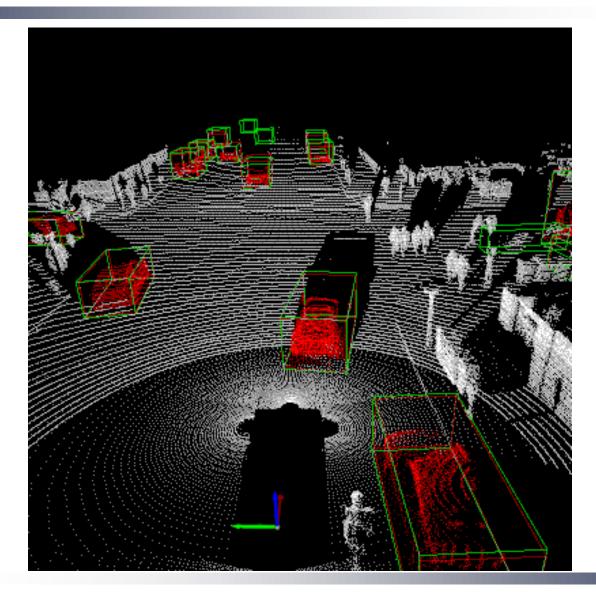




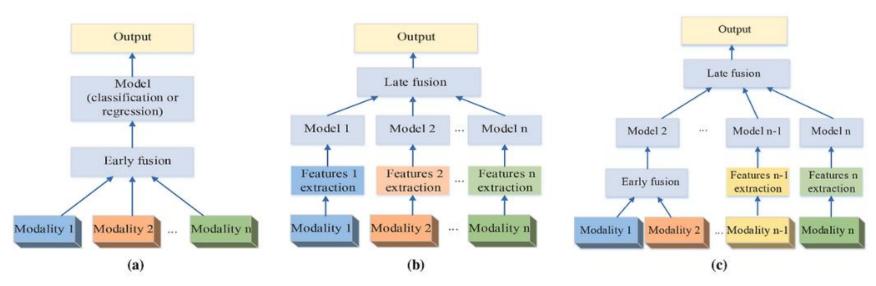
### RADAR view



### LiDAR view







(a) Early fusion (data or feature-level); (b) Late fusion (decision-level); (c) Hybrid fusion.



#### Raw level fusion / Raw data fusion

- 센서에서 수집한 원시(raw) 데이터를 가공하지 않고 직접 합침.
- 센서들의 해상도, 위치 보정, 시간 동기화가 선행되어야 함.

#### ▶ 예시

- 카메라 이미지의 픽셀 위치와 LiDAR 포인트의 위치를 정렬해서 같은 위치에 표시.
- 레이더 거리 데이터 + 카메라 RGB 영상을 하나의 화면에 오버레이.
- 여러 IMU의 가속도 데이터를 평균해서 노이즈를 줄임.

#### ▶ 장점

- 정보 손실 없이 다양한 특성 결합 가능
- 복합적인 특성을 가진 입력 생성

#### ▶ 단점

- 시간/공간 정렬이 어렵고 계산량 큼
- 센서 특성 차이(해상도, 범위 등) 보정 필요



#### **Feature level fusion**

- 각 센서의 데이터를 먼저 가공해서 '특징(feature)'만 추출한 뒤, 이를 결합.
- 특징이란? → 엣지(edge), 코너, 객체 후보 박스 등

#### ▶ 예시

- 카메라에서 "사람 얼굴"이라는 객체 특징을 추출, 레이더에서 "속도" 정보를 추출 → 합쳐서 "달리는 사람" 판단
- 영상에서 차선 검출, LiDAR에서 평면 특징 검출 → 도로 형태 예측

#### > 장점

- 센서 특성 간 균형 유지
- 처리 효율이 좋음

#### ▶ 단점

- 중요한 정보가 feature 추출 단계에서 손실될 수 있음
- 특징 추출 알고리즘의 성능에 의존



#### **Decision level fusion**

• 각 센서가 독립적으로 판단을 내린 후, 그 판단을 결합해서 최종 결정.

#### ▶ 예시

- 카메라가 "차 있음" 판단 + 레이더도 "차 있음" 판단 → 최종적으로 "앞에 차 있음" 확정
- 차량 주행 중: GPS는 "위치 정확", IMU는 "직진 중" → 판단 일치 시 신뢰도 상승

#### > 장점

- 구현이 간단하고 유연성 있음
- 센서 간 독립적 유지 가능

#### ▶ 단점

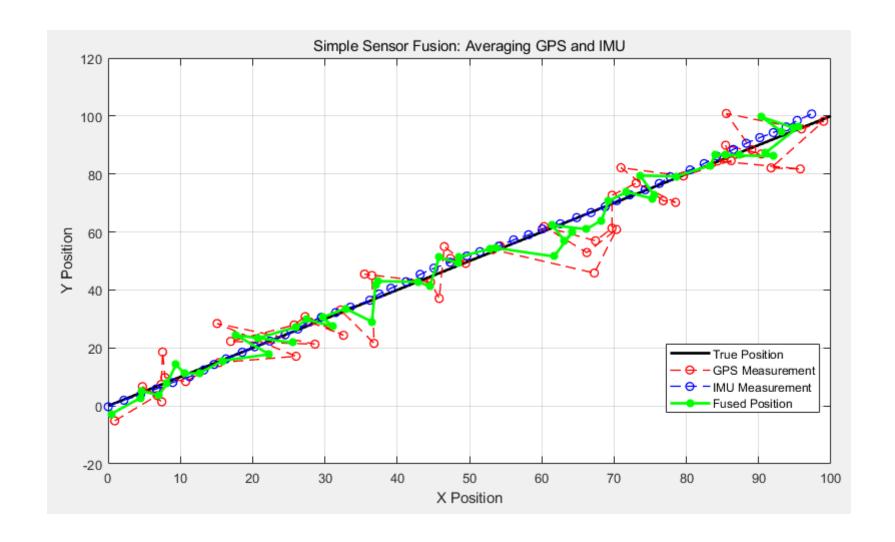
- 정보 손실 큼 (센서가 판단한 '결과'만 활용)
- 결정 오류 시 수정 어려움

https://www.youtube.com/watch?v=wKNvzLgTYhQ

https://www.youtube.com/watch?v=ZauJbv4AdWI



### GPS+IMU



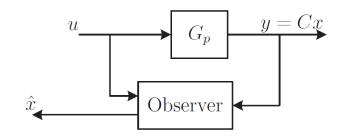


### Camera + RADAR



### State observer

#### **State Observer:**



$$G_p: \begin{cases} x(k+1) = \Phi x(k) + \Gamma u(k) \\ y(k) = Cx(k) \end{cases}$$

State observer

$$\hat{x}(k+1) = \Phi\hat{x}(k) + \Gamma u(k) + L(y(k) - C\hat{x}(k))$$

Estimation error  $e(k) = x(k) - \hat{x}(k)$ 

$$e(k+1) = \Phi e(k) - LCe(k) = (\Phi - LC)e(k)$$
 state observation error dynamics

The observer gain L can be chosen such that  $e \to 0$ , irrespective of u (provided it's known and used)



### State observer

#### **Prediction-Correction State Observer:**

An LTI system

$$G_p: \begin{cases} x(k+1) = \Phi x(k) + \Gamma u(k) \\ y(k) = Cx(k) \end{cases}$$

Prediction:

$$\bar{x}(k+1) = \Phi \hat{x}(k) + \Gamma u(k)$$

Correction:

$$\hat{x}(k) = \bar{x}(k) + L(y(k) - C\bar{x}(k))$$

Estimation error  $\bar{e}(k) = x(k) - \bar{x}(k)$ 

$$\bar{e}(k+1) = (\Phi - \Phi LC)\bar{e}(k)$$
 state observation error dynamics

Note:  $\bar{e}(\cdot)$  convergent  $\Rightarrow e(\cdot) = x(\cdot) - \hat{x}(\cdot)$  convergent.



### Kalman filter: white Gaussian noise

#### **Discrete Kalman Filter**

Let us consider a discrete-time system

$$G_p: \begin{cases} x_{k+1} = \Phi_k x_k + w_k \\ y_k = C_k x_k + v_k \end{cases}$$

where

 $w_k$  : white sequence with known covariance,  $~~ m{\sim} ~ N(0,Q_k)$ 

 $v_k$ : white sequence measurement error with known covariance,  $~~ \sim ~ N(0,R_k)$ 

The covariance matrices for the  $w_k$  and  $v_k$ 

$$\mathbb{E}[w_k w_k^T] = Q_k, \quad \mathbb{E}[w_k w_j^T] = 0 \quad (j \neq k)$$

$$\mathbb{E}[v_k v_k^T] = R_k, \quad \mathbb{E}[v_k v_j^T] = 0 \quad (j \neq k)$$

$$\mathbb{E}[w_k v_i^T] = 0 \quad (\forall k, j)$$



Prediction (*a priori*) estimate  $\bar{x}_k$ 

Prediction (a priori) estimation error

$$\bar{e}_k = x_k - \bar{x}_k$$

Prediction (a priori) error covariance matrix

$$\bar{\Sigma}_k = \mathbb{E}[\bar{e}_k \bar{e}_k^T] = \mathbb{E}[(x_k - \bar{x}_k)(x_k - \bar{x}_k)^T]$$

How to use the measurement  $y_k$  to improve the prior estimate  $\bar{x}_k$ 



We choose

$$\hat{x}_k = \bar{x}_k + L_k(y_k - C_k \bar{x}_k)$$

where

 $\hat{x}_k$  : the updated (*a posteriori*) estimate

 $L_k$ : a gain to be determined

How to find the gain  $L_k$  that yields an updated estimate that is optimal in some sense

· minimum mean-square error as a performance criterion



Updated (a posteriori) estimation error:

$$e_k = x_k - \hat{x}_k$$

The covariance associated with the updated estimate error:

$$\Sigma_k = \mathbb{E}[e_k e_k^T] = \mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T]$$

Using

$$\hat{x}_k = \bar{x}_k + L_k(y_k - C\bar{x}_k) = (I - L_k C)\bar{x}_k + L_k Cx_k + L_k v_k$$
$$e_k = x_k - \hat{x}_k = (I - L_k C)\bar{e}_k - L_k v_k$$

we have

$$\Sigma_k = \mathbb{E}[e_k e_k^T] = (I - L_k C) \bar{\Sigma}_k (I - L_k C)^T + L_k R_k L_k^T$$

(The *a priori* estimation error  $\bar{e}_k$  uncorrelated with  $v_k$ ,  $\mathbb{E}[\bar{e}_k v_k^T] = 0$ )



### Kalman filter: optimization

#### **Optimization:**

Need to solve an optimization problem

$$\min_{L_k} \operatorname{tr}[\Sigma_k]$$

subject to Convergence

where

$$\Sigma_{k} = \mathbb{E}[e_{k}e_{k}^{T}] = (I - L_{k}C_{k})\bar{\Sigma}_{k}(I - L_{k}C_{k})^{T} + L_{k}R_{k}L_{k}^{T}$$

$$= \bar{\Sigma}_{k} - L_{k}C_{k}\bar{\Sigma}_{k} - \bar{\Sigma}_{k}C_{k}^{T}L_{k}^{T} + L_{k}(C_{k}\bar{\Sigma}_{k}C_{k}^{T} + R_{k})L_{k}^{T}$$

Good to note:

For 
$$A=[a_{kl}]\in\mathbb{C}^{n\times m}$$
,  $B=[b_{kl}]\in\mathbb{C}^{n\times m}$ , 
$$\operatorname{tr}\left[A^TB\right]=\sum_{k=1}^n\sum_{l=1}^ma_{kl}b_{kl}=a_{11}b_{11}+\cdots+a_{nm}b_{nm}$$

If 
$$A = B$$
,

$$\operatorname{tr}\left[A^{T}A\right] = \sum_{k=1}^{n} \sum_{l=1}^{m} a_{kl}^{2} = a_{11}^{2} + \dots + a_{nm}^{2}$$



### Kalman filter: optimization

$$\frac{d\operatorname{tr}[\Sigma_k]}{dL_k} = -2(C_k\bar{\Sigma}_k)^T + 2L_k(C_k\bar{\Sigma}_kC_k^T + R_k) = 0$$

Optimal gain

$$L_k = \bar{\Sigma}_k C_k^T (C_k \bar{\Sigma}_k C_k^T + R_k)^{-1}$$
 (Kalman gain)

The covariance matrix associated with the optimal estimate

$$\Sigma_k = (I - L_k C_k) \bar{\Sigma}_k (I - L_k C_k)^T + L_k R_k L_k^T$$

Substituting the optimal gain leads to

$$\Sigma_k = (I - L_k C_k) \bar{\Sigma}_k$$



Prediction model

$$\bar{x}_{k+1} = \Phi_k \hat{x}_k$$

The error covariance matrix associated with  $\bar{x}_{k+1}$ 

$$\bar{e}_{k+1} = x_{k+1} - \bar{x}_{k+1} = \Phi_k x_k + w_k - \Phi_k \hat{x}_k = \Phi_k e_k + w_k$$

The prediction error covariance matrix  $\bar{\Sigma}_{k+1} = \mathbb{E}[\bar{e}_{k+1}\bar{e}_{k+1}^T]$ 

$$\left| \bar{\Sigma}_{k+1} = \Phi_k \Sigma_k \Phi_k^T + Q_k \right|$$

 $(\mathbb{E}[e_k w_k^T] = 0, \ e_k \ \text{uncorrelated with} \ w_k)$ 



Convergence?

Assume stationary, fixed sample period process

$$\bar{\Sigma}_{k+1} = \Phi \Sigma_k \Phi^T + Q_k = (\Phi - \Phi L_k C) \bar{\Sigma}_k (\Phi - \Phi L_k C)^T + \Phi L_k R_k L_k^T \Phi^T + Q_k$$

assures

$$(\Phi - \Phi L_k C) \bar{\Sigma}_k (\Phi - \Phi L_k C)^T - \bar{\Sigma}_{k+1} + Q_k < 0$$

#### Note:

- · Steady state prediction error dynamics
- $\cdot \ \bar{\Sigma} > 0$  for the Lyapunov inequality



### Kalman filter: summary

#### **Discrete Kalman Filter Algorithm:**

• Correction update (using measurement  $y_k$ ):

$$L_k = \bar{\Sigma}_k C_k^T (C_k \bar{\Sigma}_k C_k^T + R_k)^{-1}$$
$$\hat{x}_k = \bar{x}_k + L_k (y_k - C_k \bar{x}_k)$$
$$\Sigma_k = (I - L_k C_k) \bar{\Sigma}_k$$

Prediction update:

$$\bar{x}_{k+1} = \Phi_k \hat{x}_k$$
$$\bar{\Sigma}_{k+1} = \Phi_k \Sigma_k \Phi_k^T + Q_k$$



### Kalman filter: summary

**Time invariant system**: steady state soln.

Using

$$\Sigma_k = (I - LC)\bar{\Sigma}_k$$

and

$$\bar{\Sigma}_{k+1} = \Phi \Sigma_k \Phi^T + Q$$

we obtain

$$\bar{\Sigma}_{k+1} = \Phi(I - LC)\bar{\Sigma}_k \Phi^T + Q$$

Let  $Y=\bar{\Sigma}_{\infty}$ , then

$$L = YC^T(CYC^T + R)^{-1}$$

$$Y = \Phi(I - LC)Y\Phi^T + Q$$

Substituting  $I-LC=I-YC^T(CYC^T+R)^{-1}C$  leads to

$$Y = \Phi Y \Phi^T - \Phi Y C^T (CYC^T + R)^{-1} CY \Phi^T + Q$$



#### **Extended Kalman Filter**

Most realistic robotic problems involve nonlinear functions

$$\begin{cases} x_{k+1} = f_k(x_k, u_{k+1}) + w_k \\ z_k = h_k(x_k) + v_k \end{cases}$$

The covariance matrices for  $w_k$  and  $v_k$ 

$$\mathbb{E}[w_k w_k^T] = Q_k, \quad \mathbb{E}[w_k w_j^T] = 0 \quad (j \neq k)$$

$$\mathbb{E}[v_k v_k^T] = R_k, \quad \mathbb{E}[v_k v_j^T] = 0 \quad (j \neq k)$$

$$\mathbb{E}[w_k v_i^T] = 0 \quad (\forall k, j)$$

#### Kalman Filter Revisited:

• Correction update (using measurement  $z_k$ ):

$$L_k=ar{\Sigma}_k \pmb{C}_k^T (\pmb{C}_k ar{\Sigma}_k \pmb{C}_k^T + R_k)^{-1}$$
 (Kalman gain) 
$$\hat{x}_k=\bar{x}_k + L_k (z_k - C_k ar{x}_k)$$
 
$$\Sigma_k=(I-L_k \pmb{C}_k) ar{\Sigma}_k$$

Prediction update:

$$\bar{x}_{k+1} = \Phi_k \hat{x}_k$$
$$\bar{\Sigma}_{k+1} = \Phi_k \Sigma_k \Phi_k^T + Q_k$$



**EKF Linearization:** First Order Taylor Series Expansion

Prediction:

$$f(x_{k-1}, u_k) \approx f(\hat{x}_{k-1}, u_k) + \frac{\partial f(x_{k-1}, u_k)}{\partial x_{k-1}} \Big|_{\hat{x}_{k-1}} (x_{k-1} - \hat{x}_{k-1})$$
$$= f(\hat{x}_{k-1}, u_k) + \Phi_k(x_{k-1} - \hat{x}_{k-1})$$

Correction:

$$h(x_k) \approx h(\bar{x}_k) + \frac{\partial h(x_k)}{\partial x_k} \Big|_{\bar{x}_k} (x_k - \bar{x}_k)$$
$$= h(\bar{x}_k) + C_k(x_k - \bar{x}_k)$$



#### **Extended Kalman Filter Algorithm:**

• Correction update (using measurement  $z_k$ ):

$$L_k = \bar{\Sigma}_k C_k^T (C_k \bar{\Sigma}_k C_k^T + R_k)^{-1}$$
$$\hat{x}_k = \bar{x}_k + L_k (z_k - h_k(\bar{x}_k))$$
$$\Sigma_k = (I - L_k C_k) \bar{\Sigma}_k$$

Prediction update:

$$\bar{x}_{k+1} = f_k(\hat{x}_k, u_{k+1})$$
$$\bar{\Sigma}_{k+1} = \mathbf{\Phi}_k \mathbf{\Sigma}_k \mathbf{\Phi}_k^T + Q_k$$

$$\Phi_k = \frac{\partial f(x_{k-1}, u_k)}{\partial x_{k-1}} \Big|_{\hat{x}_{k-1}}, \quad C_k = \frac{\partial h(x_k)}{\partial x_k} \Big|_{\bar{x}_k}$$

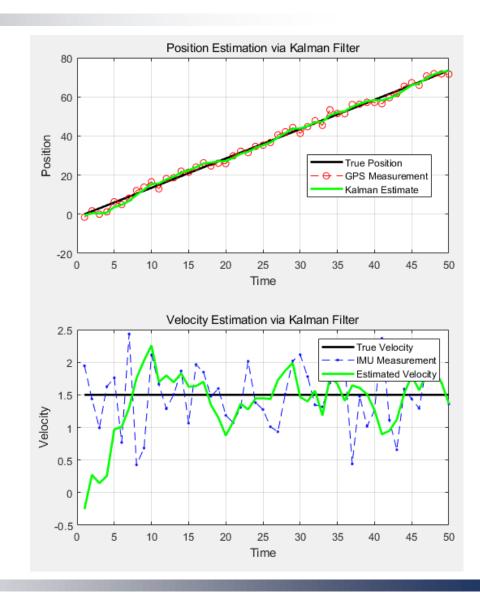


## GPS+IMU (CV)

#### 등속(등속도, Constant Velocity, CV) 모델

$$egin{bmatrix} x_{k+1} \ v_{k+1} \end{bmatrix} = egin{bmatrix} 1 & 1 \ 0 & 1 \end{bmatrix} egin{bmatrix} x_k \ v_k \end{bmatrix} + w_k$$

- 위치 업데이트:  $x_{k+1} = x_k + v_k \cdot \Delta t$
- 속도는 일정:  $v_{k+1} = v_k$



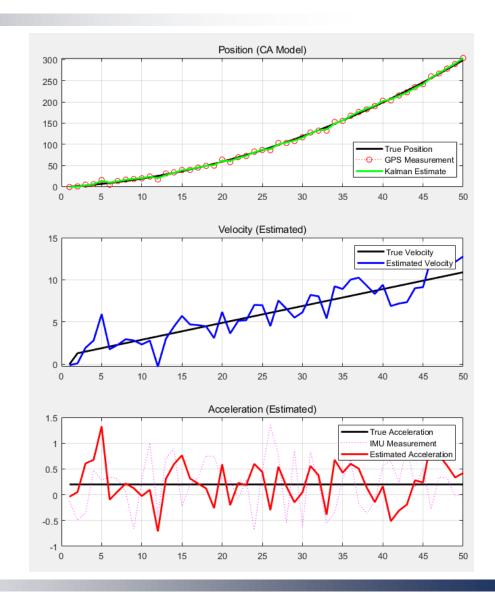


### GPS+IMU (CA)

#### 등가속도 모델(Constant Acceleration, CA)

$$x = \begin{bmatrix} ext{position} \\ ext{velocity} \\ ext{acceleration} \end{bmatrix}$$

$$A = egin{bmatrix} 1 & dt & 0.5dt^2 \ 0 & 1 & dt \ 0 & 0 & 1 \end{bmatrix}$$





### GPS+IMU (Average, CV, CA)

- 1. **단순 평균** (GPS + IMU 가속도 기반 추정)
- 2. 등속 모델 Kalman Filter
- 3. 등가속 모델 Kalman Filter

- 실제 객체는 **등가속 운동** (position + velocity + acceleration)
- GPS: noisy한 위치 측정만 제공
- IMU: noisy한 가속도 측정만 제공
- 세 가지 추정 방식으로 **동일한 참값을 기반으로 추정**하고 비교

