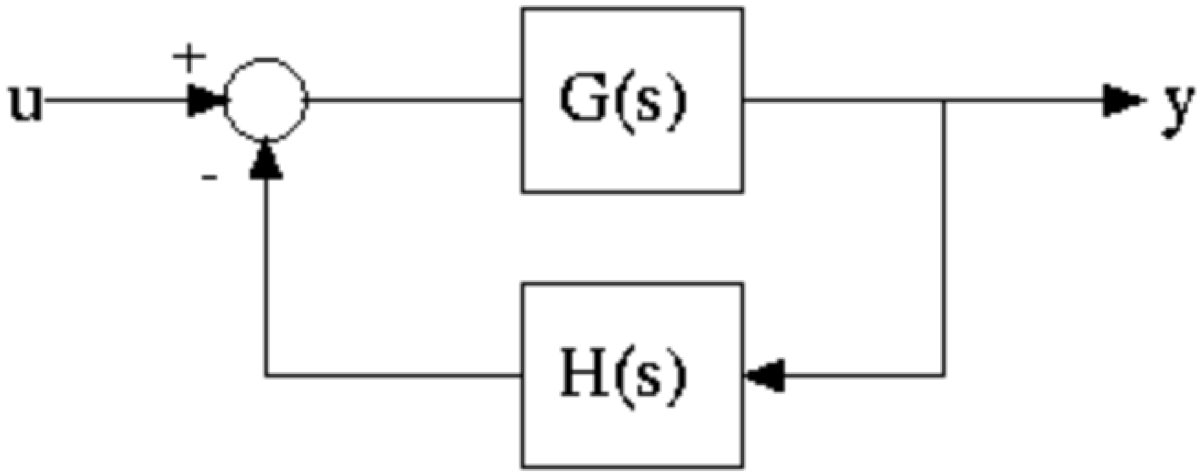
System Control

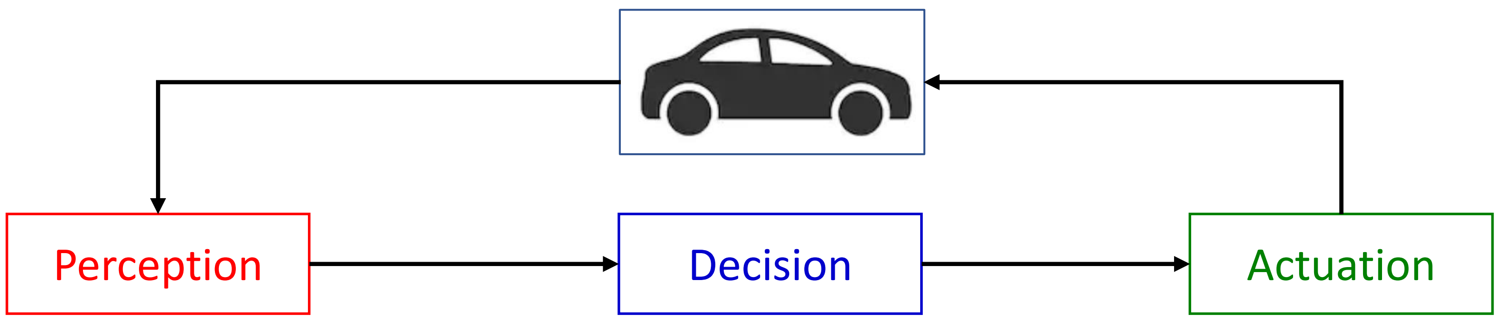
introduction



What is control?

Control = Perception + Decision + Actuation

In Feedback "Loop"



Self-Driving Car	Camera LIDAR Radar	Computer	Control Electronics
Human Driven Car	Eyes Ears	Brain	Hands Feet

What is system?

Transfer function

$$\frac{1}{ms^2 + bs + k}$$

Linear

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu + B_\phi \phi_d$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ b_{21} \\ 0 \\ b_{41} \end{bmatrix}, \; B_\phi = \begin{bmatrix} 0 \\ a_{24} - V_x \\ 0 \\ a_{44} \end{bmatrix}$$

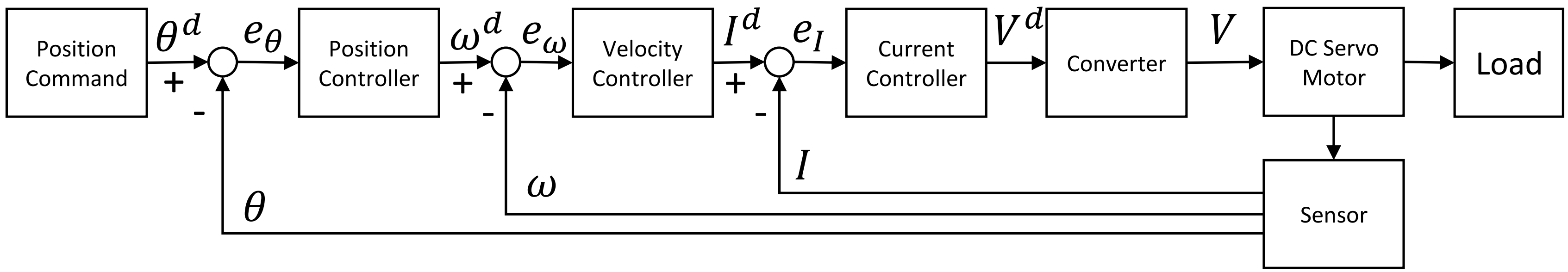
Nonlinear

$$\mathbf{x}(k+1) = f(\mathbf{x}(k), u(k), \phi(k))$$

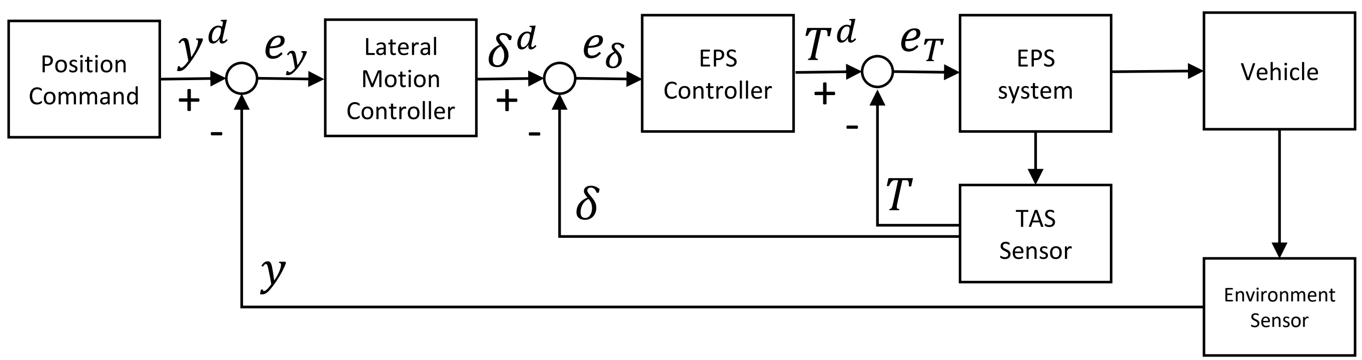
$$= \begin{bmatrix} X(k) + TV\cos(\beta(k) + \psi(k)) \\ Y(k) + TV\sin(\beta(k) + \psi(k)) \\ V(k) + Ta(k) \\ \beta(k) + T\psi(k) - T\dfrac{a_y(k)}{V_x(k)} \\ \psi(k) + T\psi(k) \\ \psi(k) - Ta_{42}\beta(k) + Ta_{44}\psi(k) + Tb_{41}\delta(k) \end{bmatrix}$$

Example of system control

Motor: position control (ex: Robot Arm)

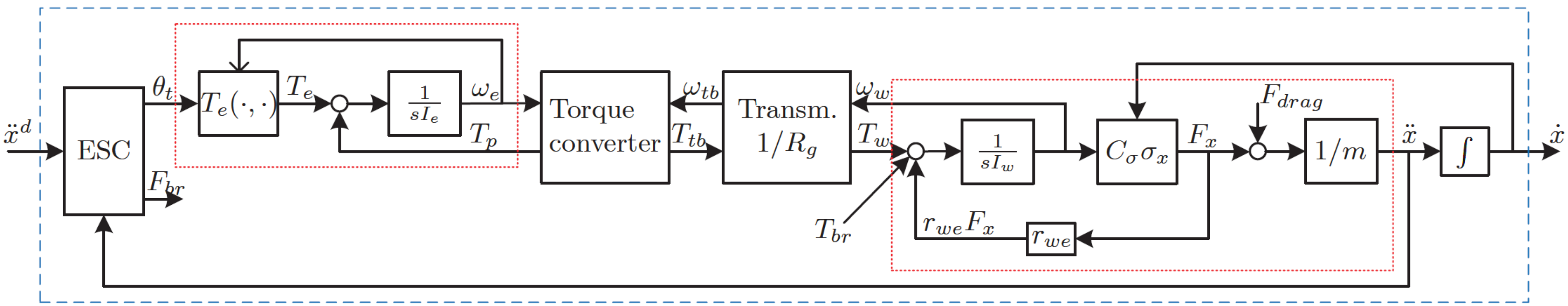


Vehicle: lateral motion control (ex: Lane Keeping System)

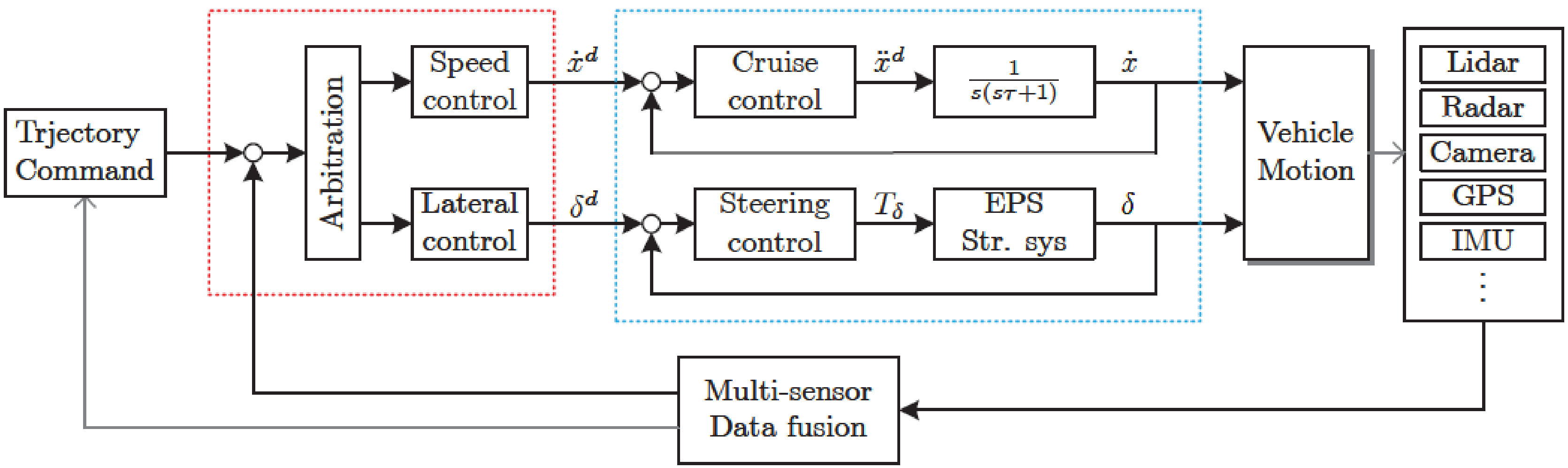


Example of system control

Longitudinal motion control

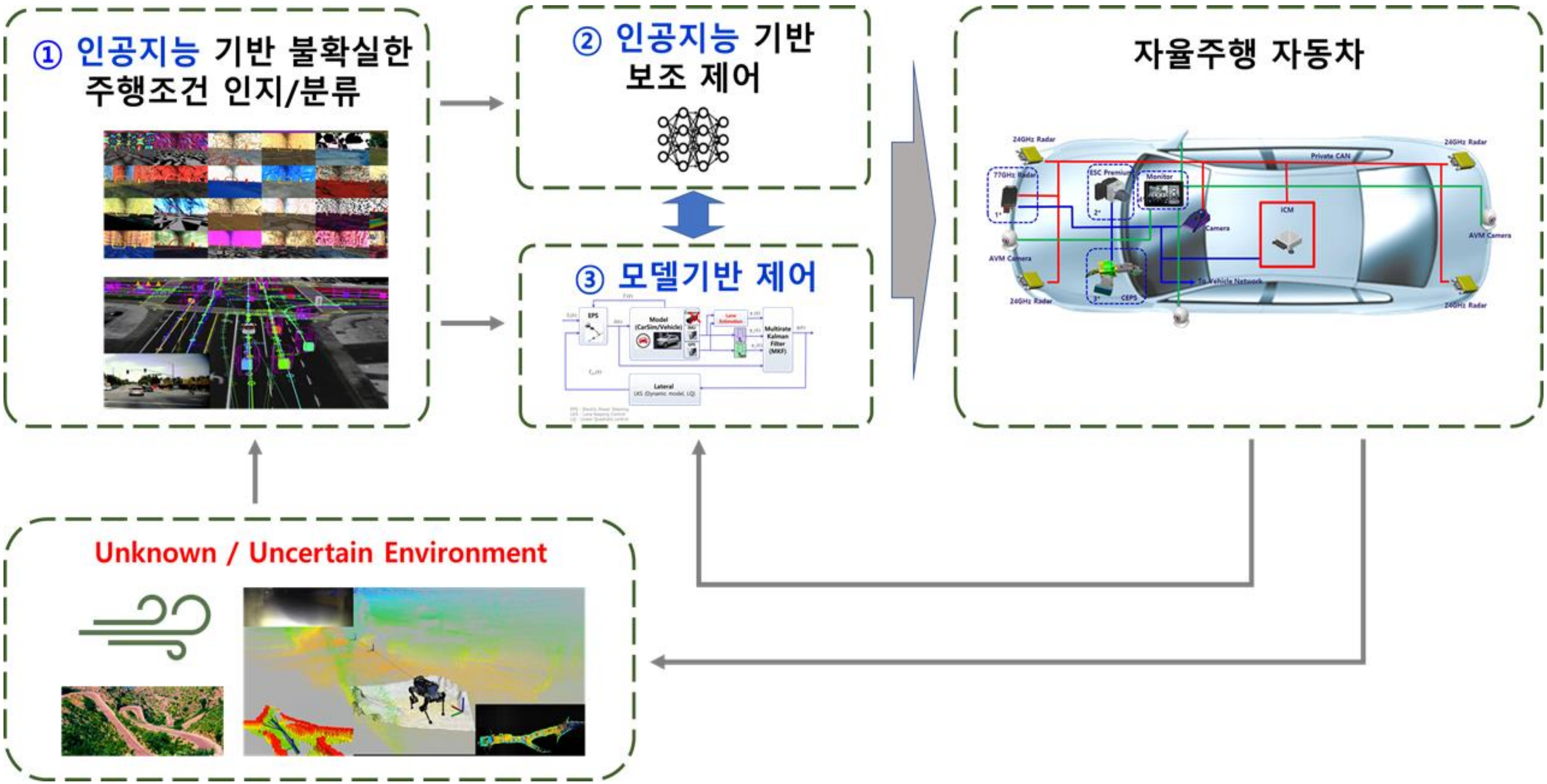


Autonomous vehicle



Example of system control

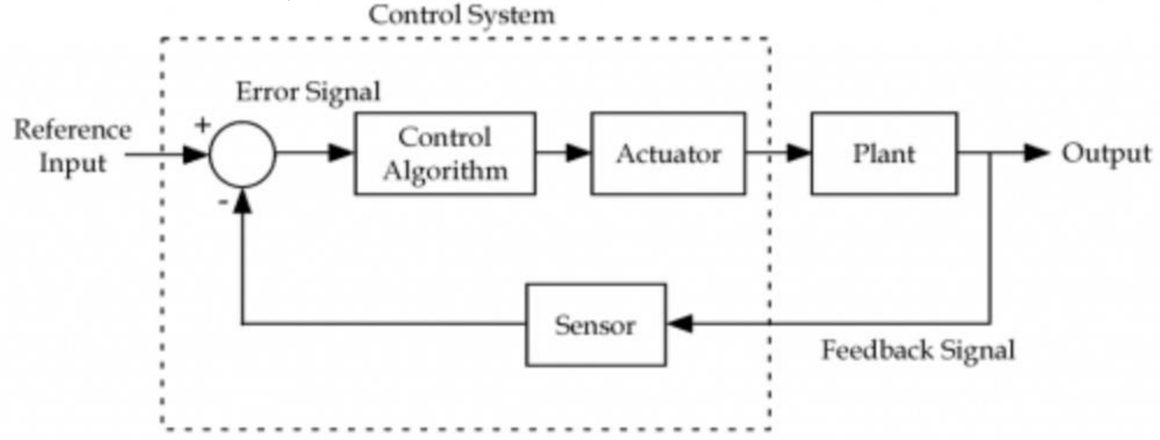
Autonomous vehicle



What is our goal?

Design a controller for given system!

- System
  - Modeling, characteristics, stability, steady-state error
- Control
  - Root locus, Digital control, …



Laplace Transform review

- The Laplace transform

$$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$$

$$s = \sigma + j\omega$$

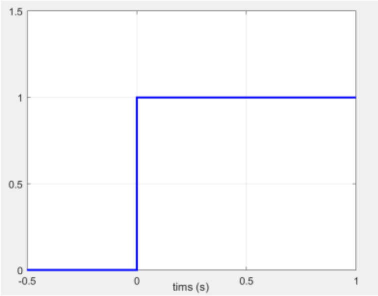
- The inverse Laplace transform

$$\mathscr{L}^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega} F(s)e^{st}ds = f(t)u(t)$$

u(t) : Unit step function

u(t) = 1   t > 0

u(t) = 0   t < 0



Laplace Transform review

- Laplace transform table

Item no.	f(t)	F(s)
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$

Item no.	f(t)	F(s)
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Laplace Transform review

- Example 2.1
  - Find the Laplace transform of

$$f(t) = Ae^{-at}u(t)$$

  - Sol)

$$F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty Ae^{-at}e^{-st}dt = A\int_0^\infty e^{-(s+a)t}dt$$

$$= -\frac{A}{s+a}e^{-(s+a)t}\Big|_{t=0}^{\infty} = \frac{A}{s+a}$$

Laplace Transform review

- Example 2.2
  - Find the inverse Laplace transform of

$$F(s) = \frac{1}{(s+3)^2}$$

  - Sol)

$$F(s) = \frac{1}{s^2} \longrightarrow f(t) = tu(t)$$

$$F(s+a) = \frac{1}{(s+a)^2} \longrightarrow f(t) = e^{-at}tu(t)$$

$$f(t) = e^{-3t}tu(t)$$

Laplace Transform review

- Partial – fraction expansion

  - Case 1. Roots of the denominator of F(s) are real and distinct

  - Case 2. Roots of the denominator of F(s) are real and repeated

  - Case 3. Roots of the denominator of F(s) are complex or imaginary

Laplace Transform review

- Case 1. Roots of the den of F(s) are real and distinct

$$F(s) = \frac{2}{(s+1)(s+2)}$$

$$F(s) = \frac{2}{(s+1)(s+2)} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+2)}$$

- To fine K_1, we first multiply (s+1)

$$\frac{2}{(s+2)} = K_1 + \frac{(s+1)K_2}{(s+2)} \qquad K_1 = 2, K_2 = -2$$

- Inverse Laplace transform

$$f(t) = (2e^{-t} - 2e^{-2t})u(t)$$

Laplace Transform review

- Case 2. Roots of the den of F(s) are real and repeated

$$F(s)=\frac{2}{(s+1)(s+2)^2}$$

  - We can write the partial – fraction expansion as a sum of terms.

$$F(s)=\frac{2}{(s+1)(s+2)^2}=\frac{K_1}{(s+1)}+\frac{K_2}{(s+2)^2}+\frac{K_3}{(s+2)}$$

  - $K_1 = 2$, which can be found as previously described
  - K_2 can be isolated by multiplying $(s+2)^2$

$$\frac{2}{(s+1)}=(s+2)^2\frac{K_1}{(s+1)}+K_2+(s+2)K_3$$

To find K_3 we see that if we differentiate with respect to s,

$$\frac{-2}{(s+1)}=\frac{(s+2)s}{(s+1)}K_1+K_3$$

$$f(t)=2e^{-t}-2te^{-2t}-2e^{-2t}$$

Laplace Transform review

- Case 3. Roots of the den of F(s) are complex or imaginary

$$F(s)=\frac{3}{s(s^2+2s+5)}$$

- This function can be expanded in the following form

$$\frac{3}{s(s^2+2s+5)}=\frac{K_1}{s}+\frac{K_2 s+K_3}{s^2+2s+5}$$

- K_1 is found in the usual way to be 3/5.

$$3=\left(K_2+\frac{3}{5}\right)s^2+\left(K_3+\frac{6}{5}\right)s+3$$

- Balancing coefficients, $\left(K_2+\frac{3}{5}\right)=0$ and $\left(K_3+\frac{6}{5}\right)=0$.

Laplace Transform review

- Case 3. Roots of the den of F(s) are complex or imaginary

$$F(s) = \frac{3}{s(s^2+2s+5)} = \frac{3/5}{s} - \frac{3}{5}\frac{s+2}{s^2+2s+5}$$

$$F(s) = \frac{3/5}{s} - \frac{3}{5}\frac{(s+1)+(1/2)(2)}{(s+1)^2+2^2}$$

- Inverse Laplace transform of each term

$$f(t) = \frac{3}{5} - \frac{3}{5}e^{-t}\left(\cos 2t + \frac{1}{2}\sin 2t\right)$$

Laplace Transform review

- Laplace transform solution of a differential equation
  - Given the following differential equation, solve for y(t) if all initial conditions are zero. Use the Laplace transform.

$$\frac{d^2y}{dt^2}+12\frac{dy}{dt}+32y=32u(t)$$

  - Sol) Laplace transform

$$s^2Y(s)+12sY(s)+32Y(s)=\frac{32}{s}$$

$$Y(s)=\frac{32}{s(s^2+12s+32)}=\frac{32}{s(s+4)(s+8)}$$

$$Y(s)=\frac{32}{s(s+4)(s+8)}=\frac{K_1}{s}+\frac{K_2}{(s+4)}+\frac{K_3}{(s+8)}$$

Laplace Transform review

- Laplace transform solution of a differential equation

$$K_1 = \left.\frac{32}{(s+4)(s+8)}\right|_{s\to 0} = 1$$

$$K_2 = \left.\frac{32}{s(s+8)}\right|_{s\to -4} = -2$$

$$K_3 = \left.\frac{32}{s(s+4)}\right|_{s\to -8} = 1$$

- Hence ,

$$Y(s) = \frac{1}{s} - \frac{2}{(s+4)} + \frac{1}{(s+8)}$$

$$y(t) = (1 - 2e^{-4t} + e^{-8t})u(t)$$

Laplace Transform review

- A general linear, time-invariant differential equation

- $$a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \dots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \dots + b_0 r(t)$$
- c(t) is the output, r(t) is the input.
- Taking the Laplace transform both sides,

$$a_n s^n C(s) + a_{n-1} s^{n-1} C(s) + \dots + a_0 C(s) + \text{initial condition terms involving } c(t)$$
$$= b_m s^m R(s) + b_{m-1} s^{m-1} R(s) + \dots + b_0 R(s) + \text{initial condition terms involving r}(t)$$

Laplace Transform review

- A general linear, time-invariant differential equation
  - If we assume that all initial conditions are zero

$$(a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0)C(s) = (b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0)R(s)$$

  - The ratio of the transform, divided by the input transform

$$\frac{C(s)}{R(s)} = G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$

  - We call this ratio, G(s), the transfer function and evaluate it with zero initial condition

$$C(s) = R(s)G(s)$$

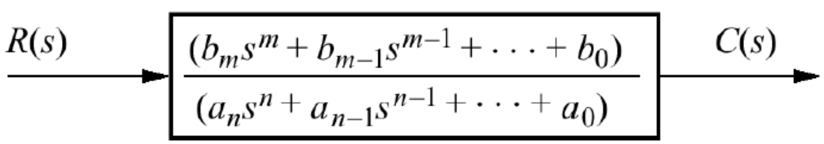


Figure Block diagram of a transfer

Laplace Transform review

- Example
  - Find the transfer function represented by

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

  - Sol) Taking the Laplace transform of both sides, assuming zero initial conditions

$$sC(s) + 2C(s) = R(s)$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

Laplace Transform review

- Example

Find the response, c(t) $\frac{dc(t)}{dt} + 2c(t) = r(t)$

- Sol) r(t)=u(t), R(s)=1/s

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

$$C(s) = G(s)R(s) = \frac{1}{s(s+2)}$$

- Expanding by partial fractions

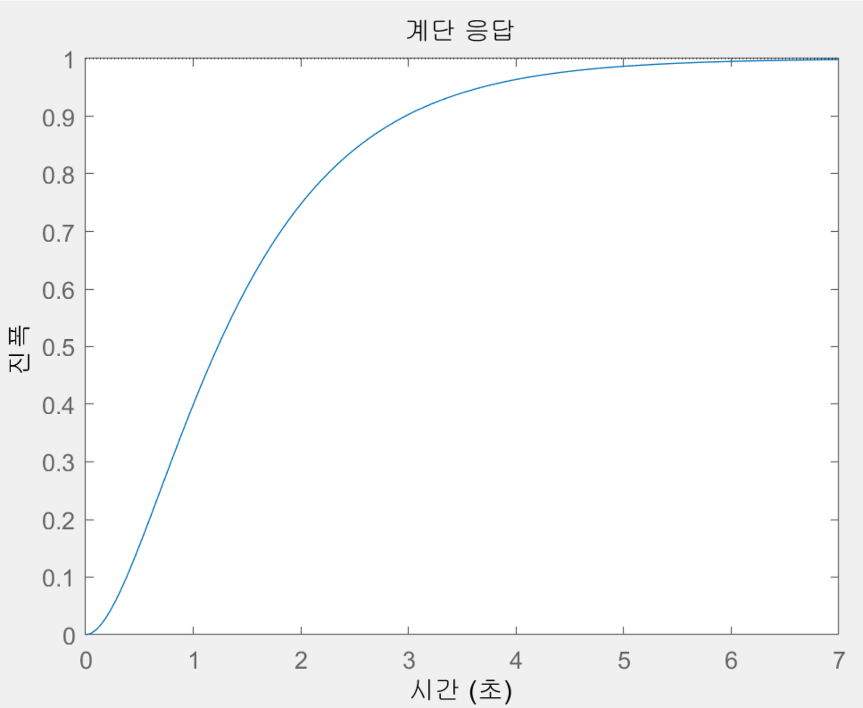
$$C(s) = \frac{1/2}{s} - \frac{1/2}{s+2}$$

- Taking inverse Laplace transform of each term yields

$$c(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

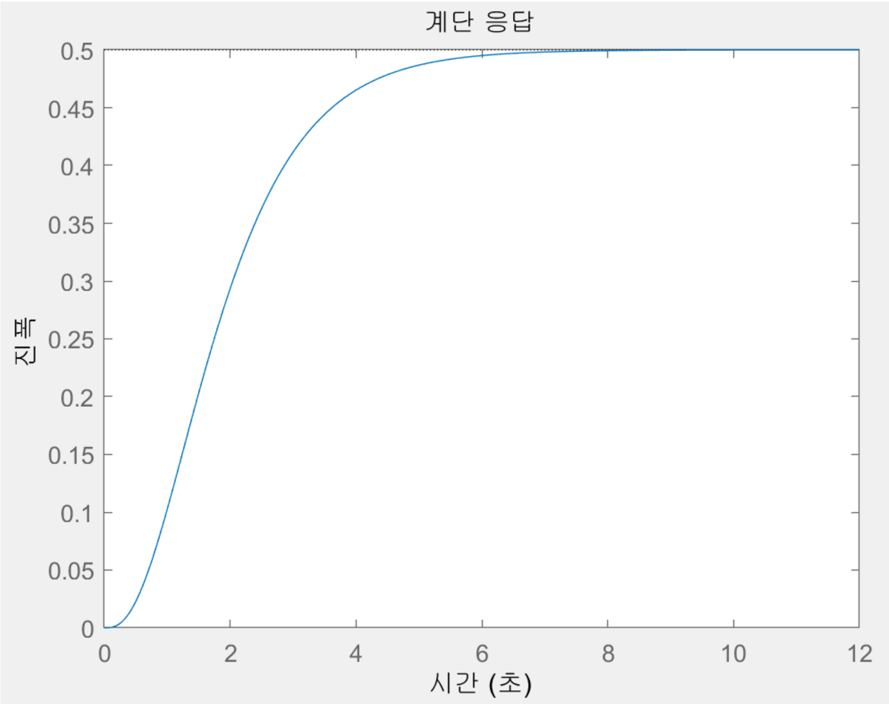
Laplace Transform review

- Case 1. vs Case 2.



[Case 1.]

$$f(t) = (2e^{-t} - 2e^{-2t})u(t)$$



[Case 2.]

$$f(t) = 2e^{-t} - 2te^{-2t} - 2e^{-2t}u(t)$$

Q & A

수업관련

- 중간고사 : 10월 넷째주
- 기말고사 : 12월 셋째주
- 출석 : LMS

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