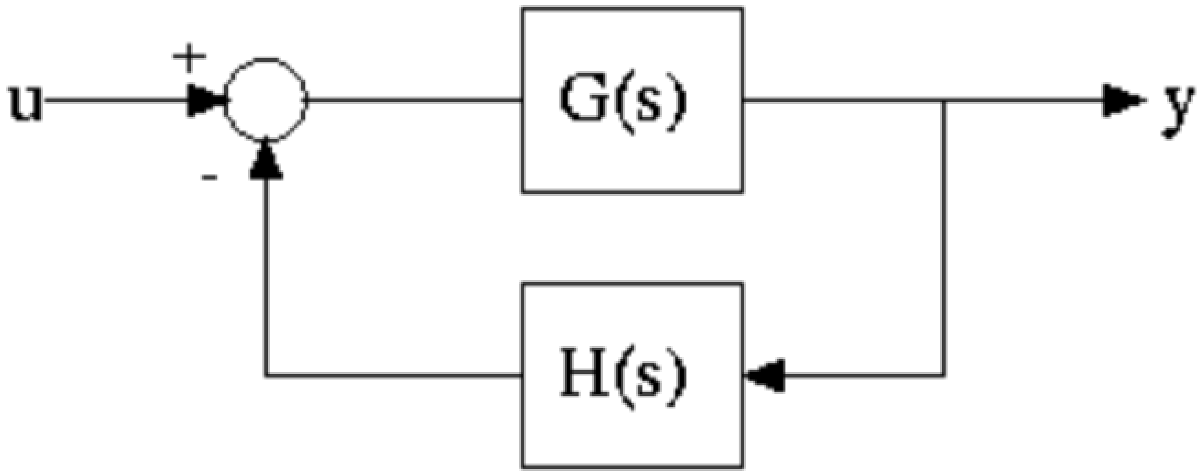
System Control

modeling in the frequency domain



System modeling

- System

Electrical system: Ohm's Law, Kirchhoff's Law

Mechanical system: Newton's Law

Electrical system + Mechanical system

= Electromechanical system (ex: Motor)

- System modeling

Modeling in the [frequency domain / time domain

Transfer function

$$\frac{1}{ms^2 + bs + k}$$

Linear

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu + B_\phi \phi_d$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ b_{21} \\ 0 \\ b_{41} \end{bmatrix},\ B_\phi = \begin{bmatrix} 0 \\ a_{24} - V_x \\ 0 \\ a_{44} \end{bmatrix}$$

Nonlinear

$$\mathbf{x}(k+1) = f(\mathbf{x}(k), u(k), \phi(k))$$
$$= \begin{bmatrix} X(k) + TV\cos(\beta(k) + \psi(k)) \\ Y(k) + TV\sin(\beta(k) + \psi(k)) \\ V(k) + Ta(k) \\ \beta(k) + T\psi(k) - T\frac{a_y(k)}{V_x(k)} \\ \psi(k) + T\psi(k) \\ \psi(k) - Ta_{42}\beta(k) + Ta_{44}\psi(k) + Tb_{41}\delta(k) \end{bmatrix}$$

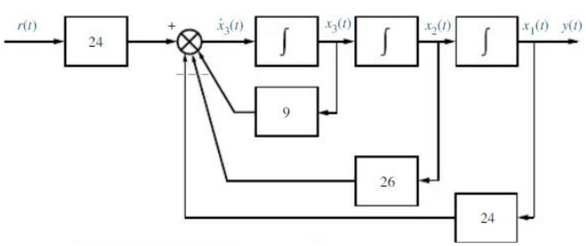
Why do we study "Laplace Transform"?

- Easy to represent and analysis a complex differential equation

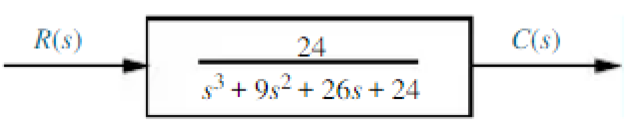
  - Represent : Transfer function
  - Analysis : Stability and transient response

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

$$\frac{d^2c}{dt^2} + 6\frac{dc}{dt} + 2c = 2\frac{dr}{dt} + r \quad \textbf{VS} \quad G(s) = \frac{C(s)}{R(s)} = \frac{2s+1}{s^2+6s+2}$$



VS



Why do we study "Laplace Transform"?

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_0 r(t)$$

⬇ Laplace Transform

$$(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0) C(s) = (b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0) R(s)$$

⬇

$$\frac{C(s)}{R(s)} = G(s) = \frac{(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0)}$$

Example 2.4

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Example 2.6

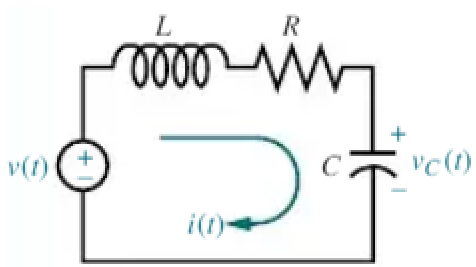
$$LC \frac{d^2 v_C(t)}{dt^2} + RC \frac{dv_C(t)}{dt} + v_C(t) = v(t)$$

Electrical system

TABLE 2.3 Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).



➔ Transfer function ??

$\frac{V_C(s)}{V(s)}$

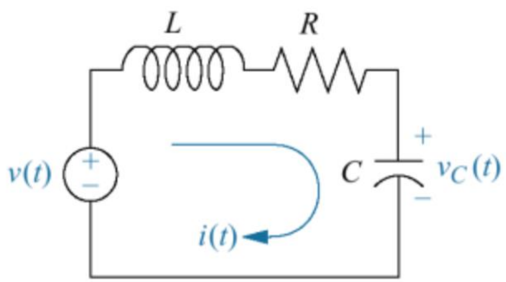
- Mesh analysis
- Nodal analysis
- Voltage division

$\frac{V_C(s)}{V(s)}$ & $\frac{I(s)}{V(s)}$

Simple Circuits via Mesh Analysis

Example

- Find the transfer function relating the capacitor voltage, $V_c(s)$, to the voltage, $V(s)$



$$L\frac{di(t)}{dt}+Ri(t)+\frac{1}{c}\int_0^t i(\tau)d\tau=v(t)$$

- Changing variables from current to charge using i(t)=dq(t)/dt yields

$$L\frac{d^2q(t)}{dt^2}+R\frac{dq(t)}{dt}+\frac{1}{c}\int_0^t q(t)dt=v(t)$$

From the voltage-charge relationship for the capacitor in Table 2.3

$$q(t)=Cv_c(s)$$

Then

$$LC\frac{d^2v_c(t)}{dt^2}+RC\frac{dv_c(t)}{dt}+v_c(t)=v(t)$$

- Taking the Laplace transform

$$\left(LCs^2+RCs+1\right)V_c(s)=V(s)$$

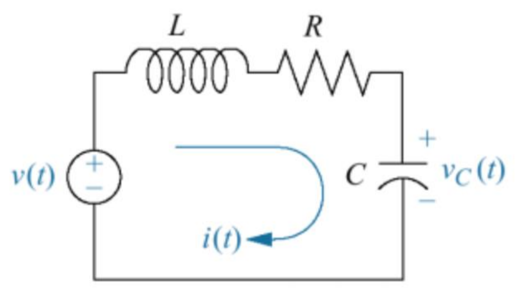
- Solving for the transfer function

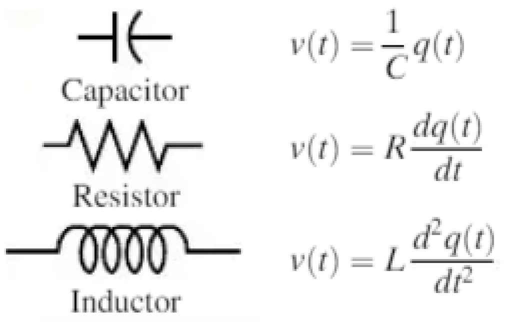
$$\frac{V_c(s)}{V(s)}=\frac{1/LC}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

Single loop via voltage division

Example

- Find the transfer function relating the capacitor voltage, $V_c(s)$, to the voltage, $V(s)$





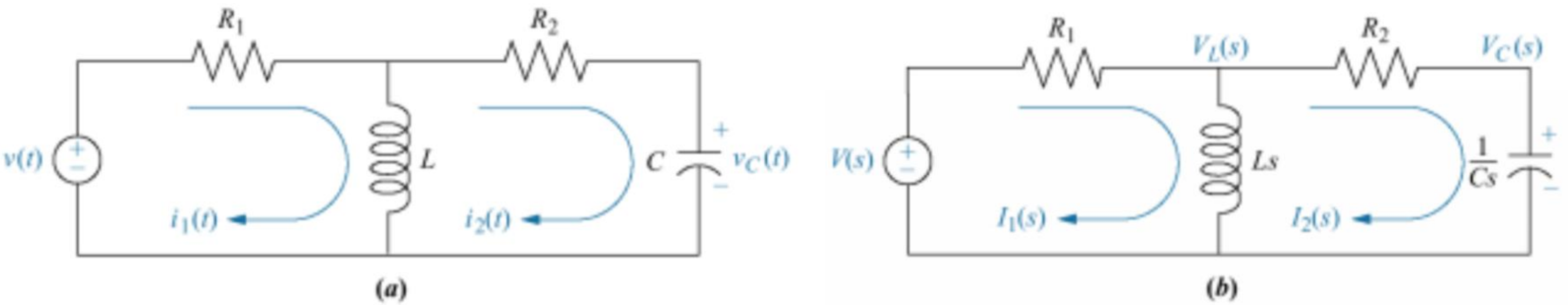
Sol) The voltage across the capacitor is some proportion of the input voltage, namely the impedance of the capacitor divided by the sum of the impedances.

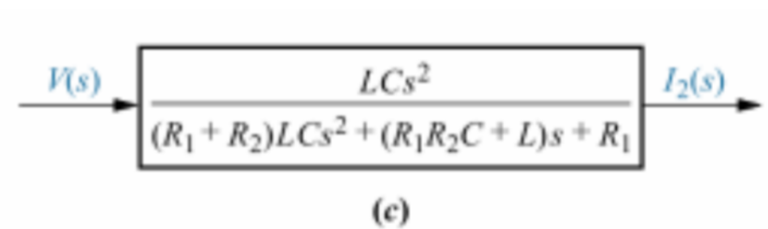
$$V_c(s) = \frac{1/Cs}{\left(Ls + R + \frac{1}{Cs}\right)}V(s)$$

Multiple loops

Example

- Find the transfer function relating the current, $I_2(s)$, to the voltage, $V(s)$



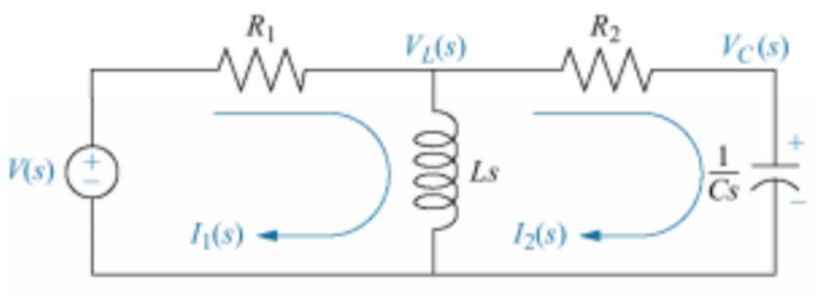


a. Two-loop electrical network ;
b. transformed two-loop electrical network;
c. block diagram;

Multiple loops

Example

- Find the transfer function relating the current, $I_2(s)$, to the voltage, $V(s)$



(b)

- Sol)

  Around Mesh1, where I1(s) flows,

$$R_1 I_1(s) + LsI_1(s) - LsI_2(s) = V(s)$$

  Around Mesh2, where I2(s) flows,

$$LsI_2(s) + R_2 I_2(s) - \frac{1}{Cs} I_2(s) - LsI_1(s) = 0$$

  combining term

$$(R_1 + Ls)I_1(s) - LsI_2(s) = V(s)$$

$$-LsI_1(s) + \left(Ls + R_2 - \frac{1}{Cs}\right)I_2(s) = 0$$

- Use Cramer's rule

$$I_2(s) = \frac{\begin{vmatrix} (R_1+Ls) & V(s) \\ -Ls & 0 \end{vmatrix}}{\Delta} = \frac{LsV(s)}{\Delta}$$

$$\Delta = \begin{vmatrix} (R_1+Ls) & -Ls \\ -Ls & \left(Ls + R_2 + \frac{1}{Cs}\right) \end{vmatrix}$$

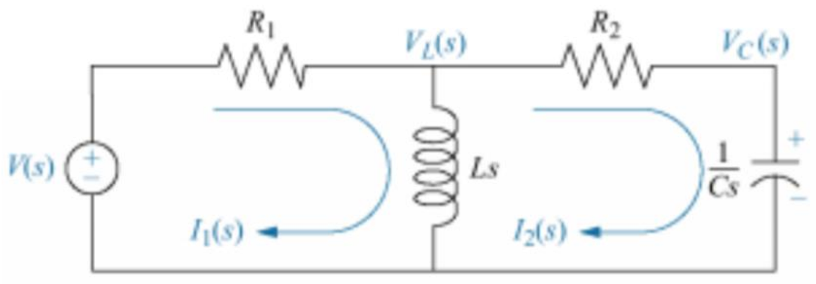
- Forming the transfer function

$$G(s) = \frac{I_2(s)}{V(s)} = \frac{Ls}{\Delta} = \frac{LCs^2}{(R_1+R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

Multiple loops

Example

- Find the transfer function relating the current, $I_2(s)$, to the voltage, $V(s)$



(b)

$$(R_1 + Ls)I_1(s) - LsI_2(s) = V(s)$$

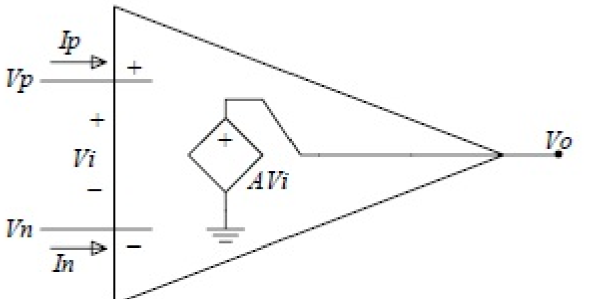
$$-LsI_1(s) + \left(Ls + R_2 - \frac{1}{Cs}\right)I_2(s) = 0$$

$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 1}\end{bmatrix} I_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to the}\\ \text{two meshes}\end{bmatrix} I_2(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{voltages around}\\ \text{Mesh 1}\end{bmatrix}$$

$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to the}\\ \text{two meshes}\end{bmatrix} I_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 2}\end{bmatrix} I_2(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{voltages around}\\ \text{Mesh 2}\end{bmatrix}$$

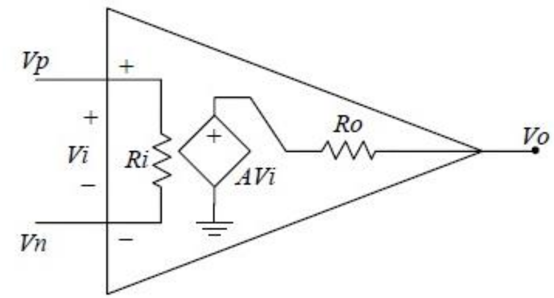
Operational Amplifiers

Ideal



1. Differential input, $V_2(t) - v_1(t)$
2. High input impedance, $Z_i = \infty$ (ideal)
3. Low output impedance, $Z_o = 0$ (ideal)
4. High constant gain amplification, $A = \infty$ (ideal)

Practical

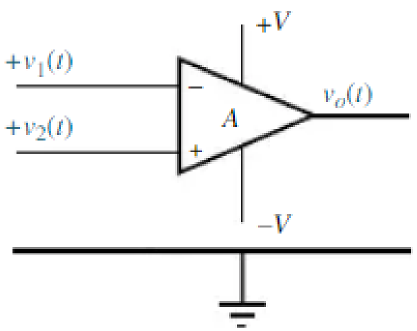


1~1000MΩ

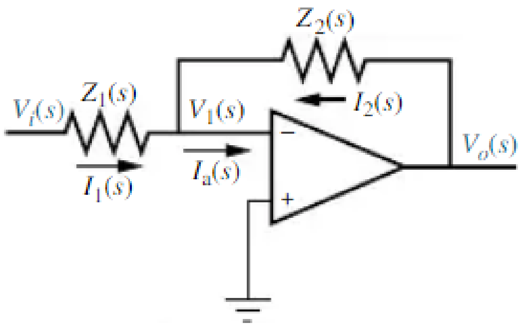
35~100Ω

100,000 ~ 1,000,000

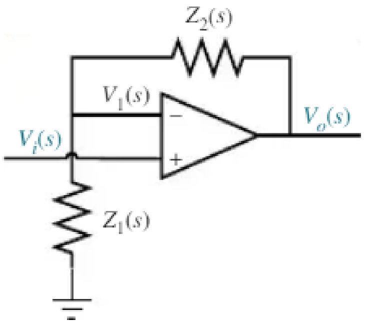
- Operational Amplifiers



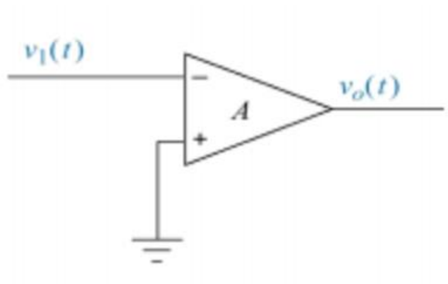
- Inverting OP amp



- Noninverting OP amp



Inverting OP amp

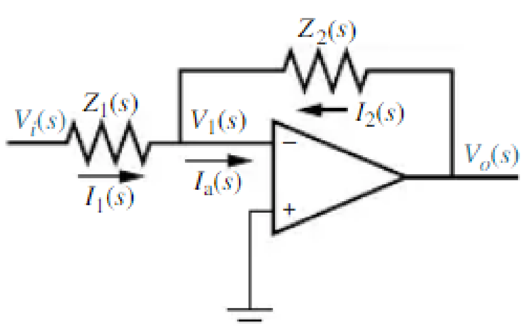


- V_2 is ground

$$V_0(t) = A(v_2(t) - v_1(t))$$

$$V_0(t) = -Av_1(t)$$

- By Kirchhoff's current law



$$I_a(s) = 0 \quad and \quad I_1(s) = -I_2(s) \quad and \quad v_1(t) \approx 0$$

$$I_1(s) = V_i(s) / Z \quad and \quad I_2(s) = V_0(s) / Z_2(s)$$

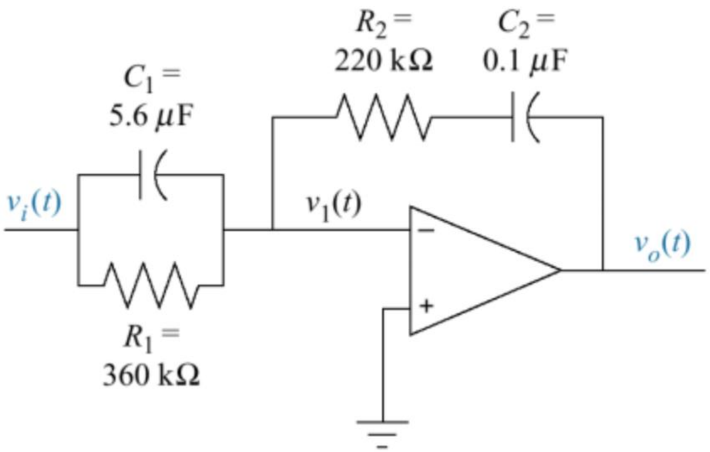
$$V_0(s) / Z_2(s) = -V_i(s) / Z_1(s)$$

$$\frac{V_0(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Inverting OP amp

Example

- Find the transfer function, $V_o(s)/V_i(s)$



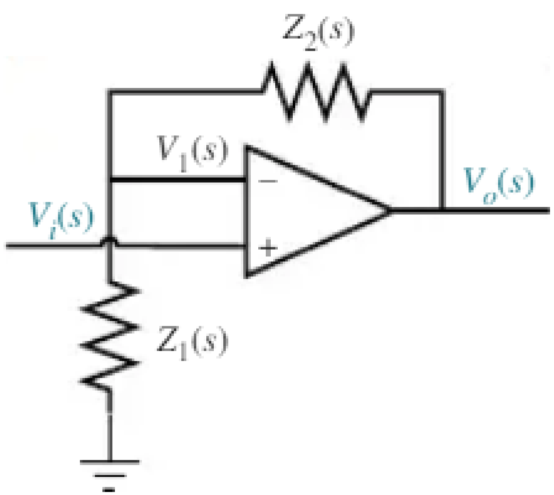
$$Z_1(s) = \frac{1}{C_1 s + \frac{1}{R_1}} = \frac{1}{5.6\times10^{-6} + \frac{1}{360\times10^3}} = \frac{360\times10^3}{2.016s+1}$$

$$Z_2(s) = R_2 + \frac{1}{Cs} = 220\times10^3 + \frac{10^7}{s}$$

$$\frac{V_0(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

$$\frac{V_o(s)}{V_i(s)} = -1.232\frac{s^2+45.95s+22.55}{s}$$

Non-Inverting OP amp



- Transfer function

$$V_o(s) = A(V_i(s) - V_1(s))$$

- Using voltage division

$$V_1(s) = \frac{Z_1(s)}{Z_1(s)+Z_2(s)} V_0(s)$$

- Substituting, rearranging

$$\frac{V_o(s)}{V_i(s)} = \frac{A}{1 + AZ_1(s)/(Z_1(s)+Z_2(s))}$$

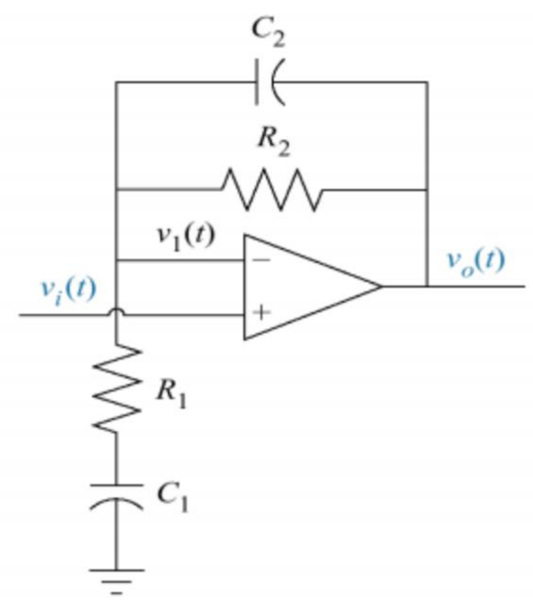
- For large A, we disregard unity in the denominator

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_1(s)+Z_2(s)}{Z_1(s)}$$

Non-Inverting OP amp

Example

- Find the transfer function, $V_o(s)/V_i(s)$



$$Z_1(s) = R_1 + \frac{1}{C_1 s}$$

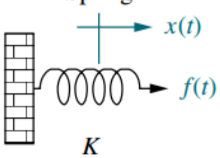
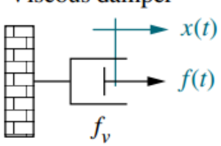
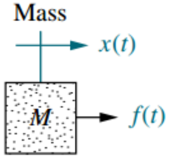
$$Z_2(s) = \frac{R_2(1/C_2 s)}{R_2 + (1/C_2 s)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_1(s) + Z_2(s)}{Z_1(s)}$$

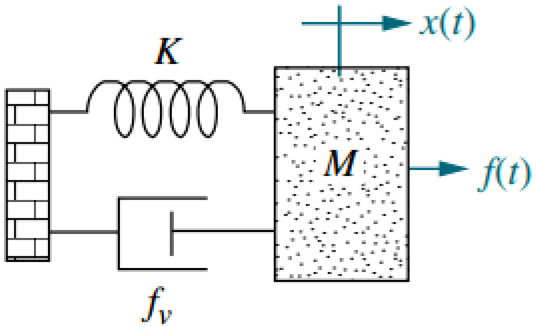
$$\frac{V_o(s)}{V_i(s)} = \frac{C_2C_1R_2R_1s^2 + (C_2R_2 + C_1R_2 + C_1R_1)s + 1}{C_2C_1R_2R_1s^2 + (C_2R_2 + C_1R_1)s + 1}$$

Mechanical system (1)

- Translational mechanical system

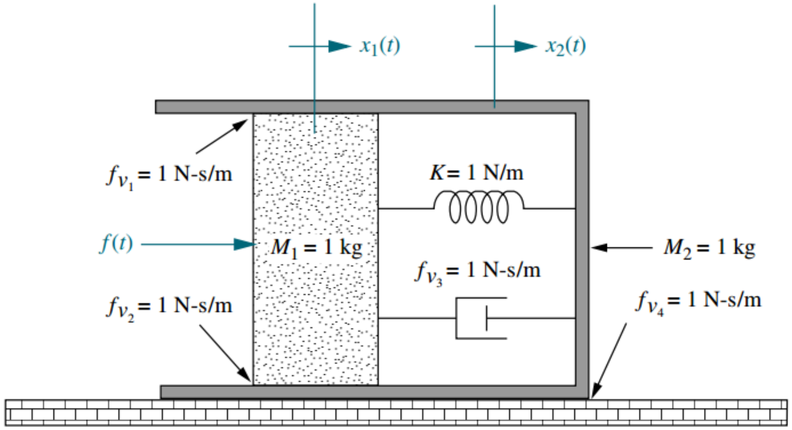
Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m(newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).



$$(Ms^2 + f_v s + K)X(s) = F(s)$$

Mechanical system (2)



- M_1 move M_2 hold
- M_1 hold M_2 move
- M_2 move M_1 hold
- M_2 hold M_1 move