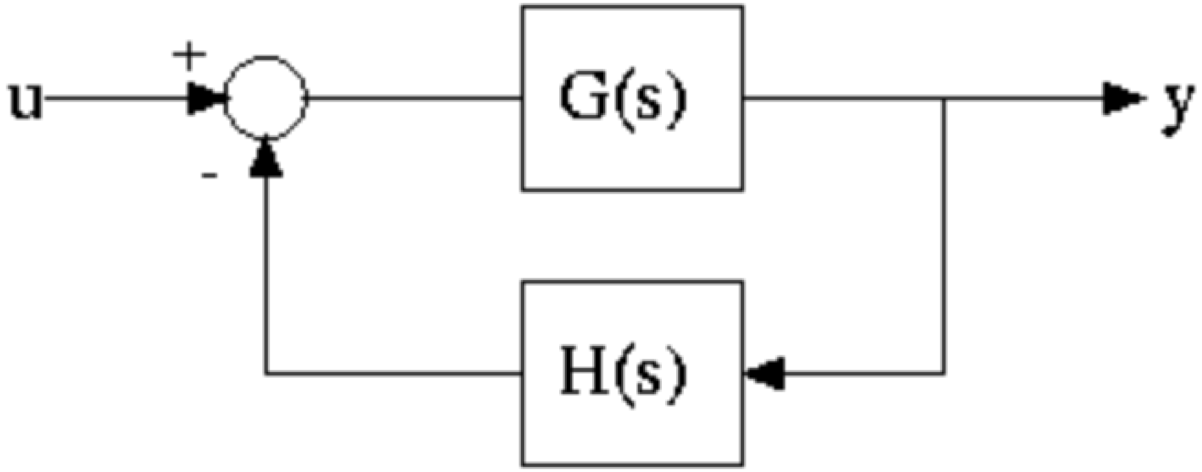
System Control

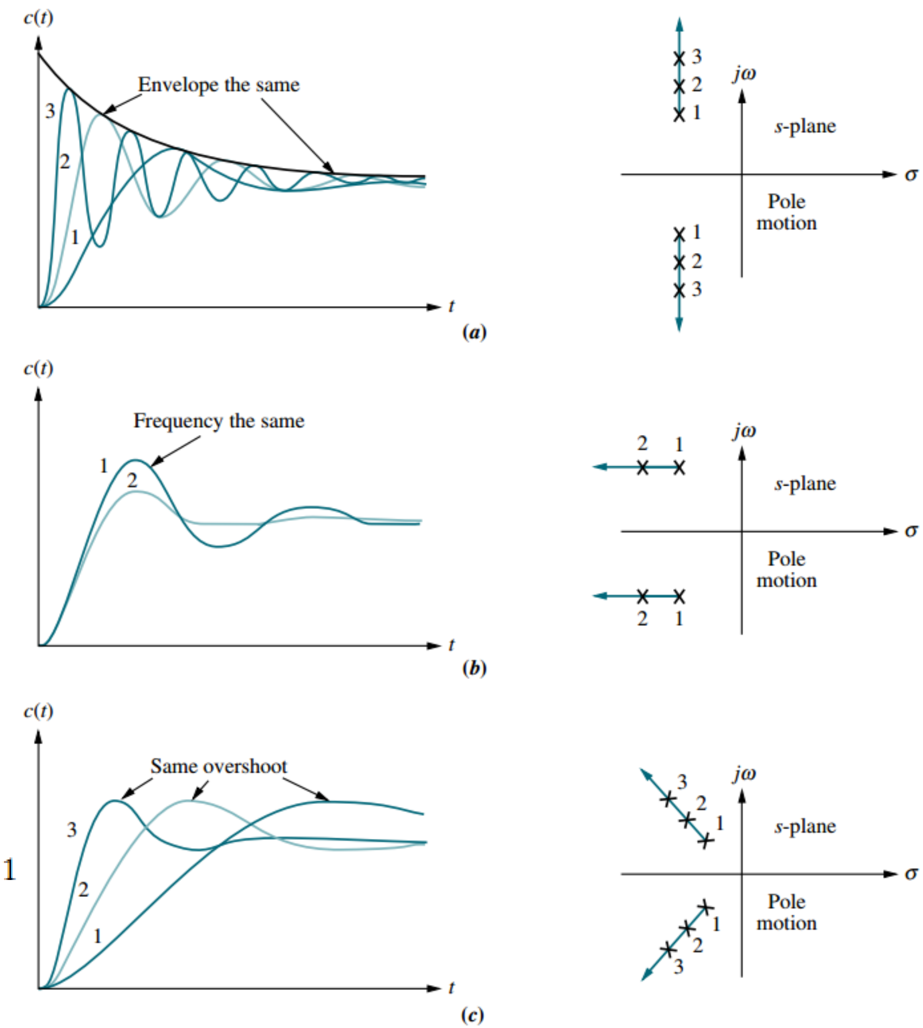
time response



Characteristics of second-order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\cos\theta = \zeta.$



1. Natural Frequency: ω_n

2. Damping ratio: ζ

3. Peak time: $T_p = \dfrac{\pi}{\omega_d}$

4. Percent overshoot:

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

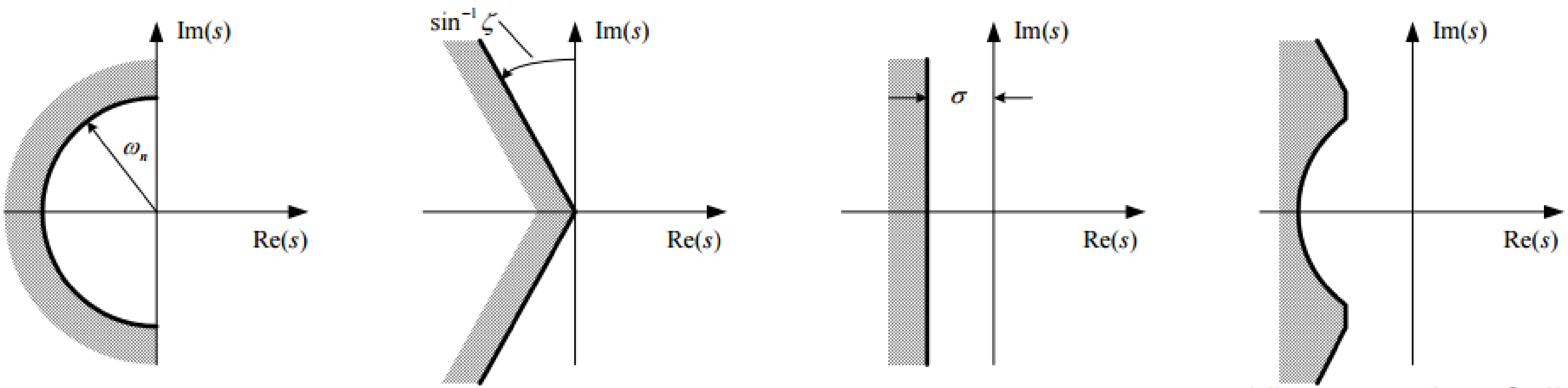
5. Rise time

$$\omega_n T_r = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1$$

6. Settling time: $T_s = \dfrac{4}{\sigma_d}$

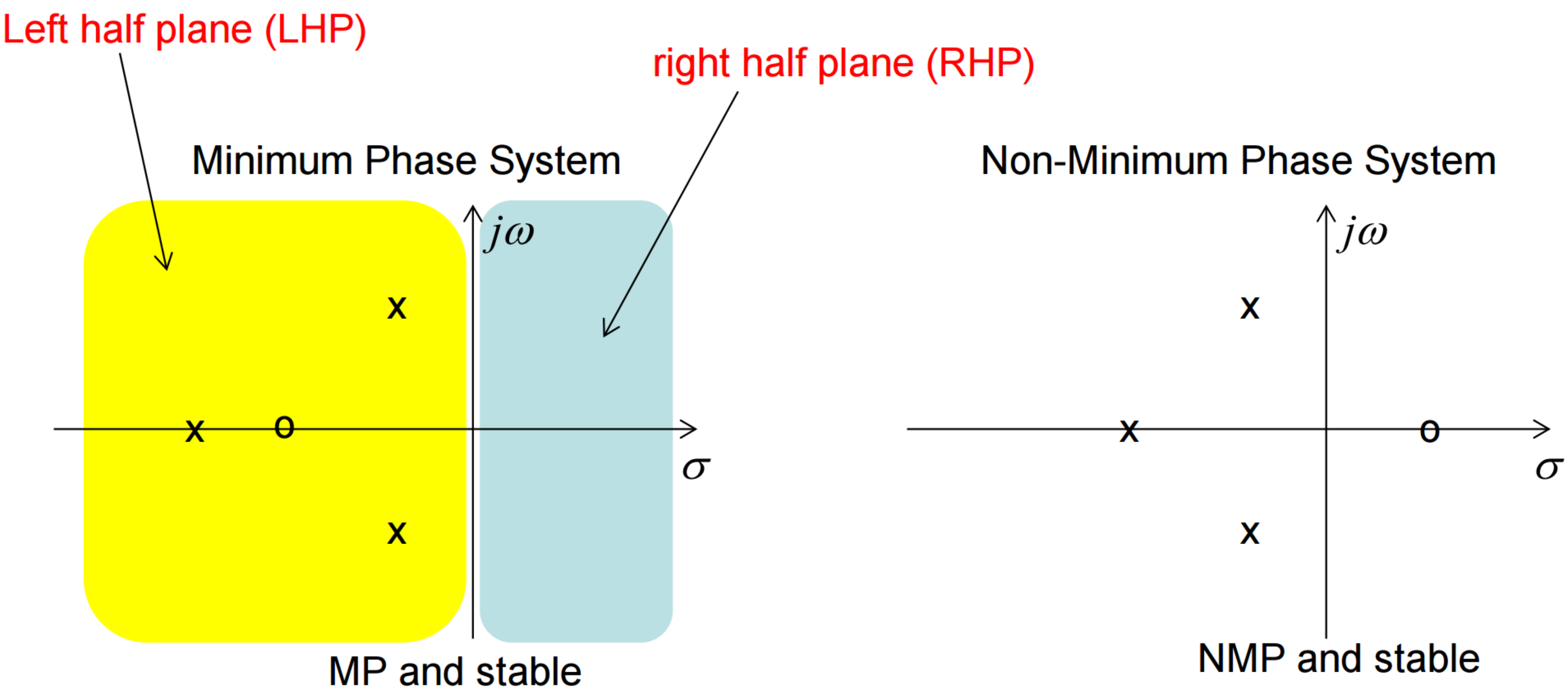
Time domain design specification

1. Peak time: $T_p = \frac{\pi}{\omega_d}$
2. Percent overshoot:
$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$
3. Rise time:
$$\omega_n T_r = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1$$
4. Settling time: $T_s = \frac{4}{\sigma_d}$



(a) rise time (b) overshoot (c) settling time (d) composite of all three requirements

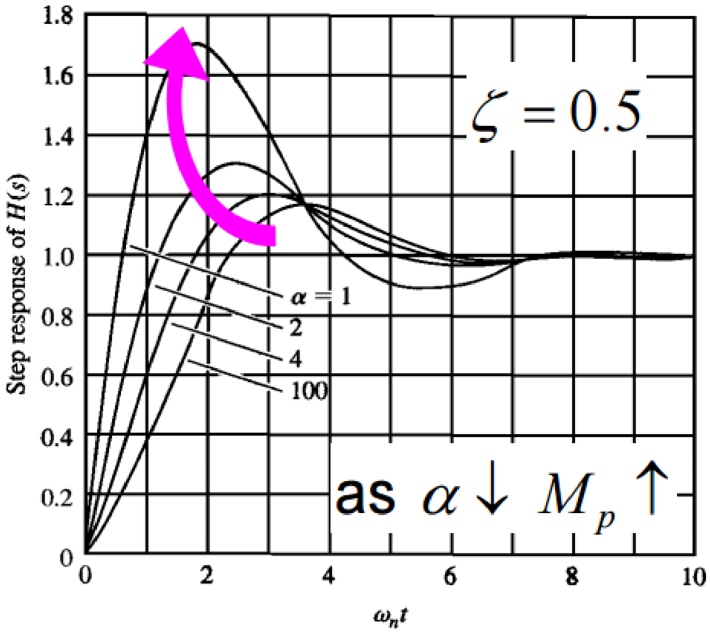
Effects of additional zero & pole

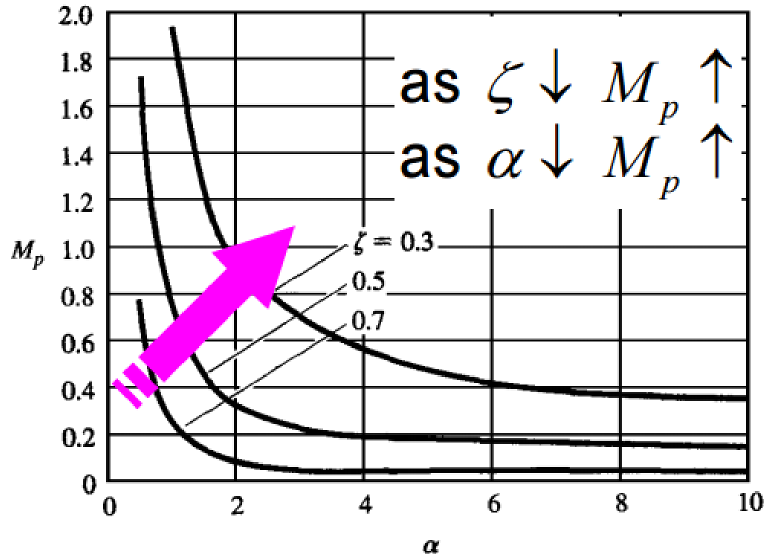


Effects of additional zero (1)

Consider $H(s) = \dfrac{(s/\alpha\zeta\omega_n)+1}{(s/\omega_n)^2 + 2\zeta(s/\omega_n)+1}$

For $\omega_n = 1$ (normalized) $H(s) = \dfrac{s/\alpha\zeta + 1}{s^2 + 2\zeta s + 1} = \underbrace{\dfrac{1}{s^2+2\zeta s+1}}_{H_o(s)} + \dfrac{1}{\alpha\zeta}\underbrace{\dfrac{s}{s^2+2\zeta s+1}}_{H_d(s)}$



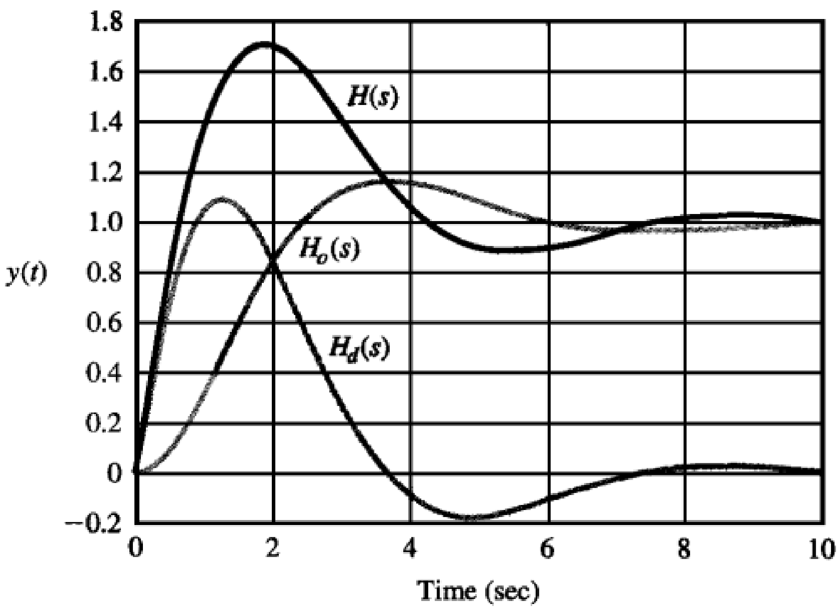


Effects of additional zero (2)

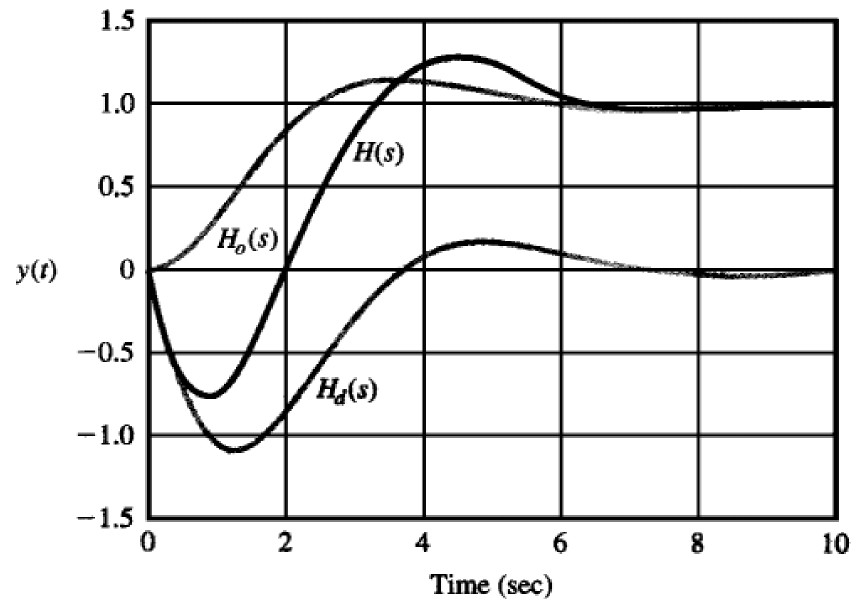
$$H(s)=\frac{s/\alpha\zeta+1}{s^2+2\zeta s+1}=\underbrace{\frac{1}{s^2+2\zeta s+1}}_{H_o(s)}+\frac{1}{\alpha\zeta}\underbrace{\frac{s}{s^2+2\zeta s+1}}_{H_d(s)}$$

(i) $\alpha > 0$ (minimum phase)

(ii) $\alpha < 0$ (nonminimum phase)



When $\alpha > 0$

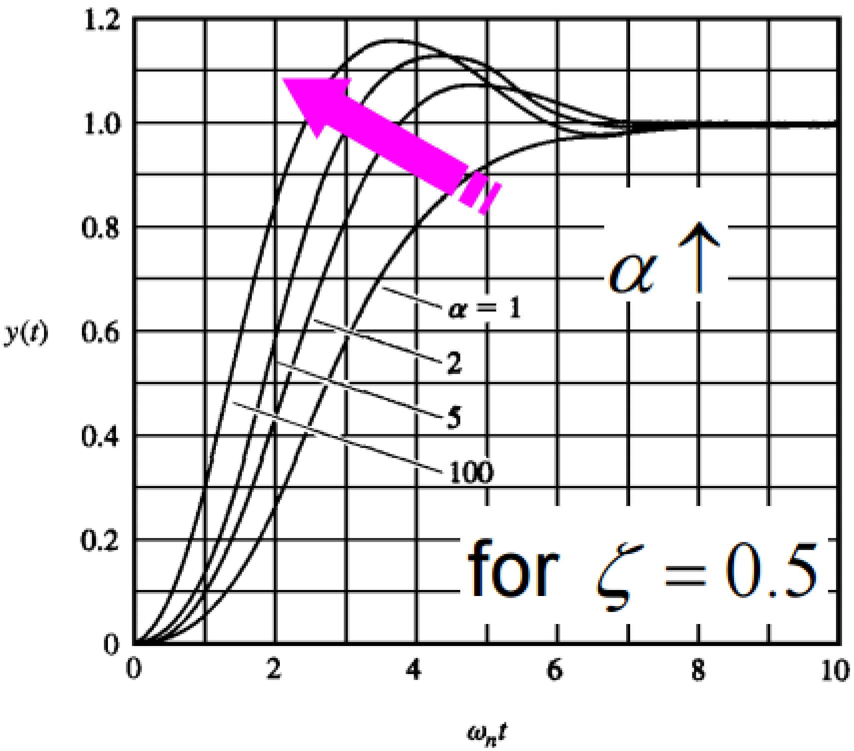


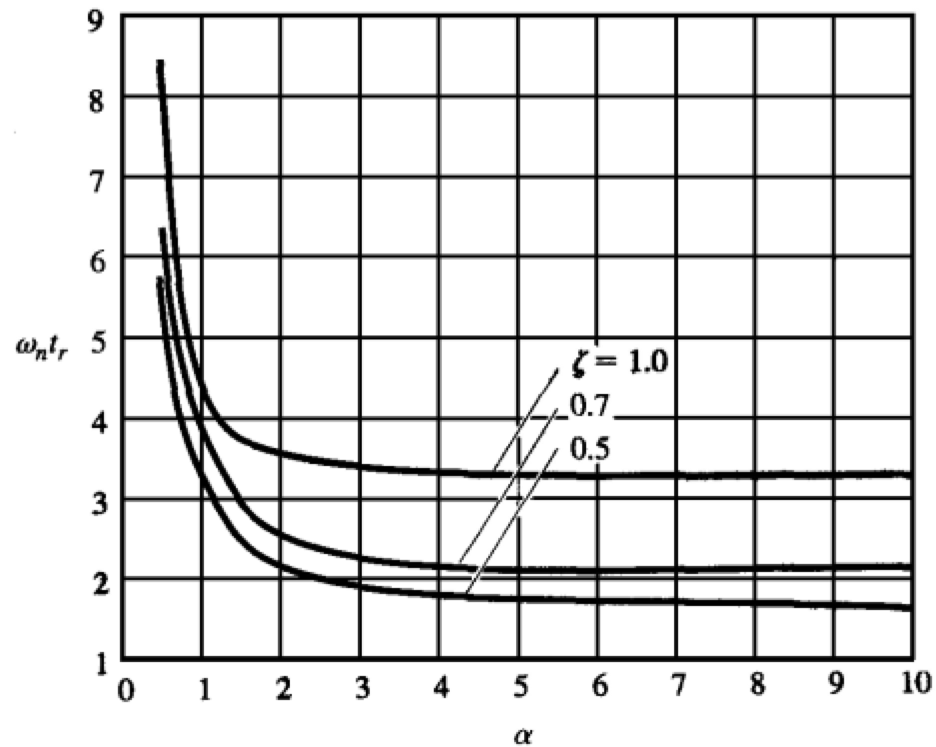
when $\alpha < 0$

※ notice that there is a undershoot to the step input

Effects of additional pole (1)

$$H(s)=\frac{1}{(s/\alpha\zeta\omega_n+1)[(s/\omega_n)^2+2\zeta(s/\omega_n)+1]}$$

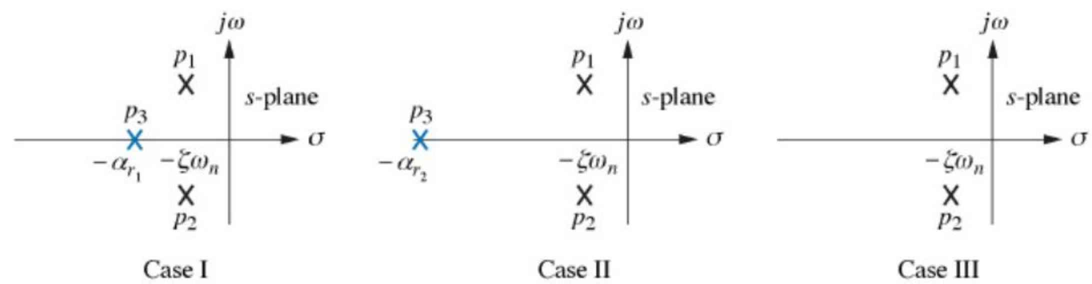




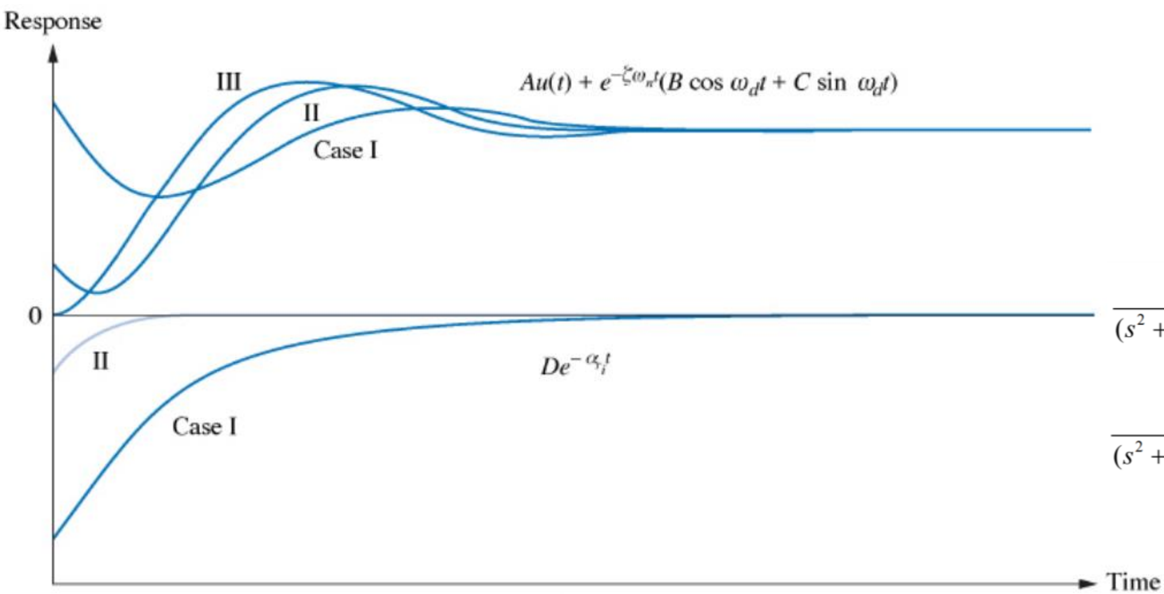
Effects of additional pole (2)

$$C(s) = \frac{bc}{s(s^2+as+b)(s+c)}$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+as+b} + \frac{D}{s+c}$$



(*a*)



(*b*)

$$\frac{1}{(s^2+4s+20)}\frac{1.5}{(s+1.5)}\frac{1}{s} = \frac{1}{20}\left(\frac{1}{s} + \frac{\alpha s+\beta}{(s^2+4s+20)} + \frac{\gamma}{(s+1.5)}\right)$$

$\gamma e^{-1.5t}$

$$\frac{1}{(s^2+4s+20)}\frac{10}{(s+10)}\frac{1}{s} = \frac{1}{20}\left(\frac{1}{s} + \frac{\alpha s+\beta}{(s^2+4s+20)} + \frac{\gamma}{(s+10)}\right)$$

γe^{-10t}

Effects of additional pole (3)

Consider again the system response:

$$C(s) = \frac{bc}{s(s^2 + as + b)(s + c)} = \frac{A}{s} + \frac{Bs + C}{s^2 + as + b} + \frac{D}{s + c}$$

We can evaluate these constants:

$$A = 1 \qquad B = \frac{ca - c^2}{c^2 + b - ca}$$

$$C = \frac{ca^2 - c^2 a - bc}{c^2 + b - ca} \qquad D = \frac{-b}{c^2 + b - ca}$$

As $c \to \infty$ $D \to 0$.

Rule of thumb: If the pole's real part is five times the real part of the dominant poles, then the system can be approximated as second-order.

Pole-zero cancellation

- **Add zero near the system pole**

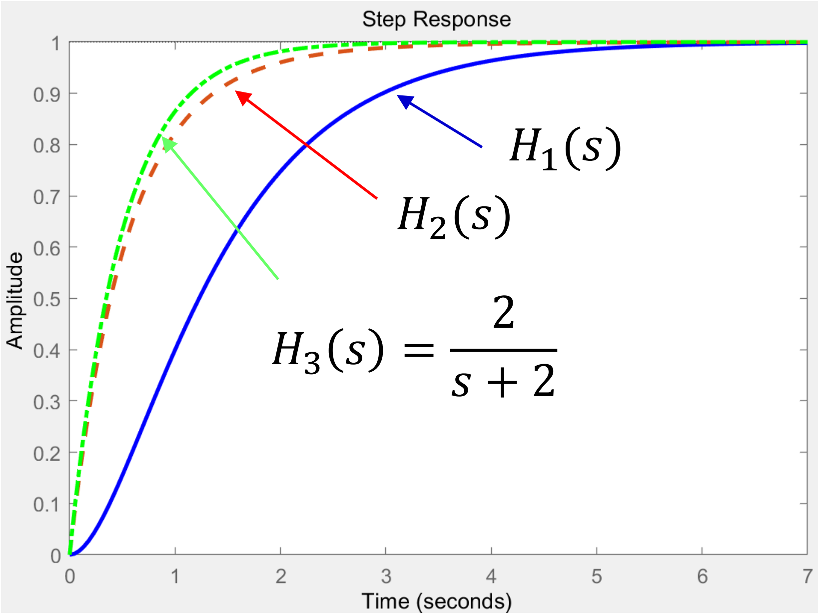
$$H_1(s) = \frac{2}{(s+1)(s+2)} = \frac{2}{s+1} - \frac{2}{s+2}$$

$$H_2(s) = \frac{2(s+1.1)}{1.1(s+1)(s+2)}$$

Close!

$$\boxed{H_1(0) = H_2(0)}$$ ⇐ same DC gain

but $\boxed{\frac{2}{s+1}} \Rightarrow \boxed{\frac{0.18}{s+1}}$

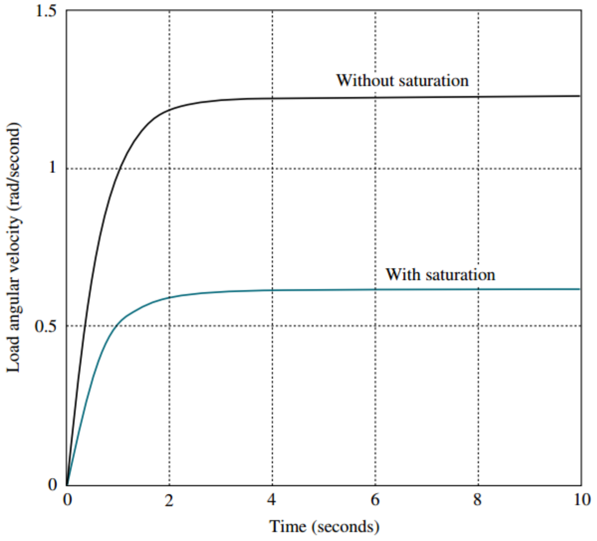


- **Consider following systems**

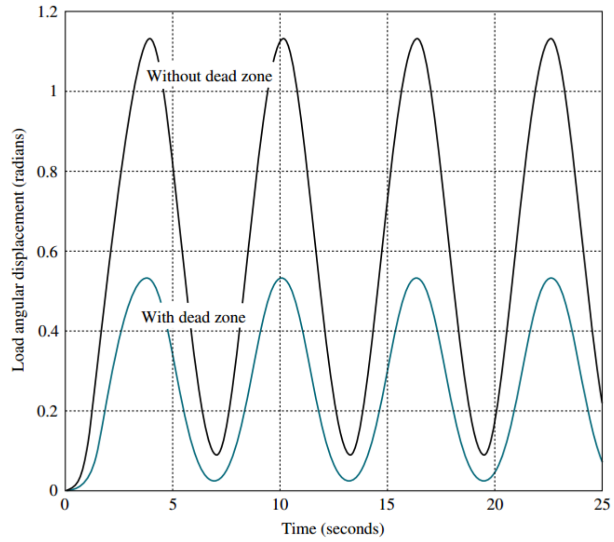
$$C_1(s) = \frac{26.25(s+4)}{s(s+3.5)(s+5)(s+6)} = \frac{1}{s} - \frac{3.5}{s+5} + \frac{3.5}{s+6} - \frac{1}{s+3.5}$$

$$C_2(s) = \frac{26.25(s+4)}{s(s+4.01)(s+5)(s+6)} = \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} - \frac{0.033}{s+4.01}$$

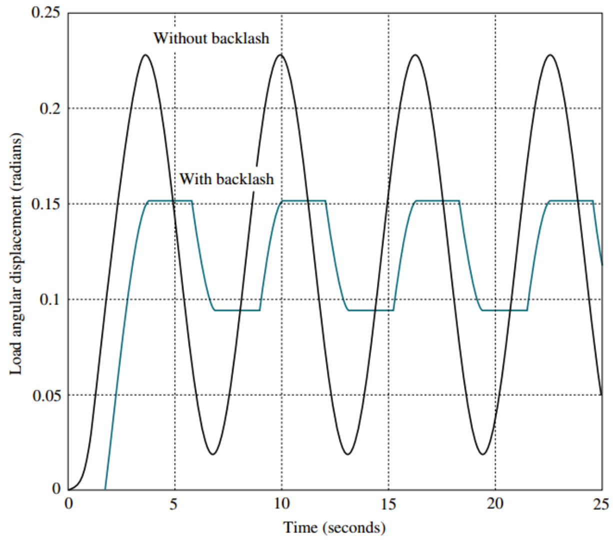
Effects of nonlinearites



saturation



dead zone



backlash