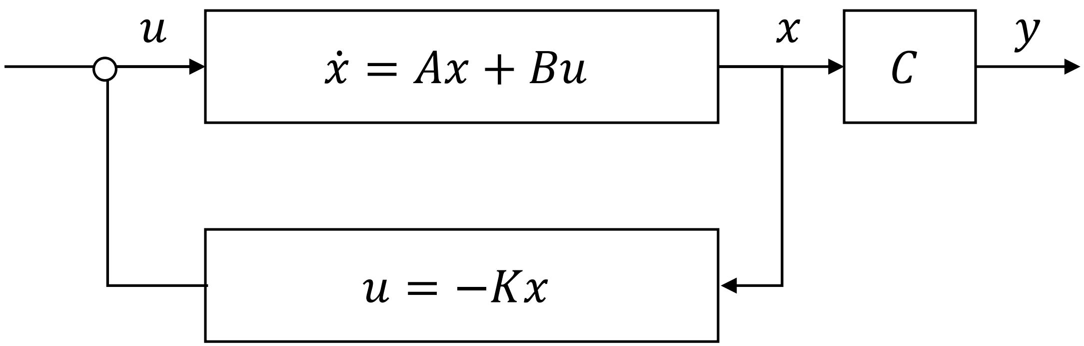
Modern Control Theory

introduction



Review : Modeling

- System

Electrical system: Ohm's Law, Kirchhoff's Law

Mechanical system: Newton's Law

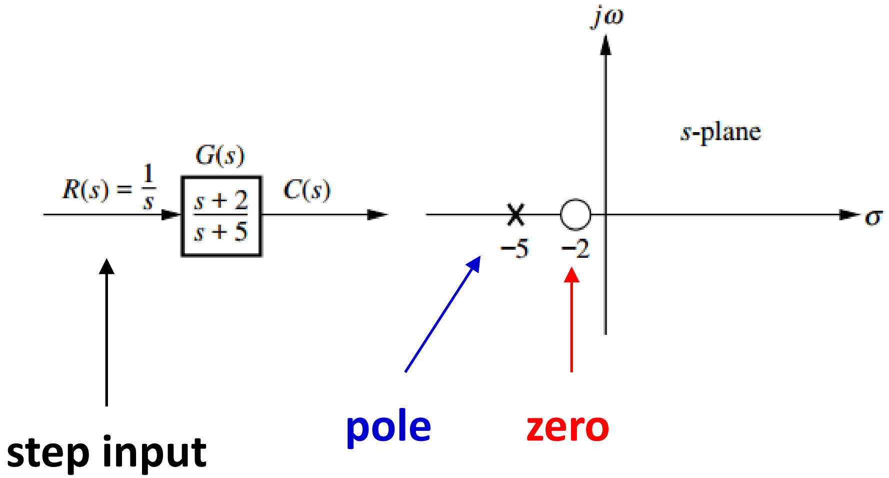
Electrical system + Mechanical system

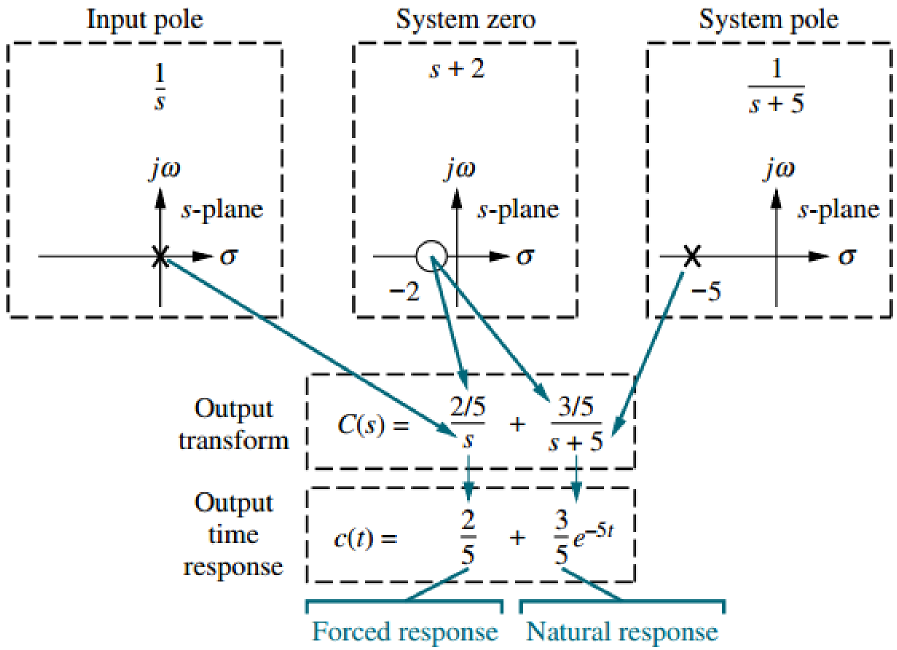
= Electromechanical system (ex: Motor)

- System modeling

Modeling in the [frequency domain
time domain

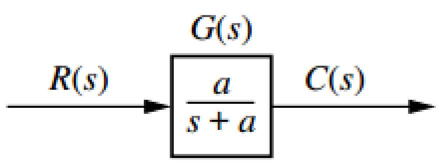
Review : Poles, zeros, and system response

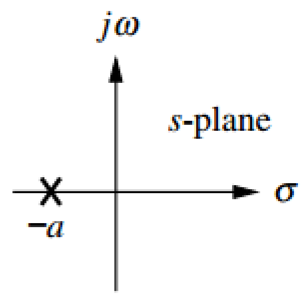




- Input poles: determine the form of the steady state response
- System poles: determines the form of the transient response

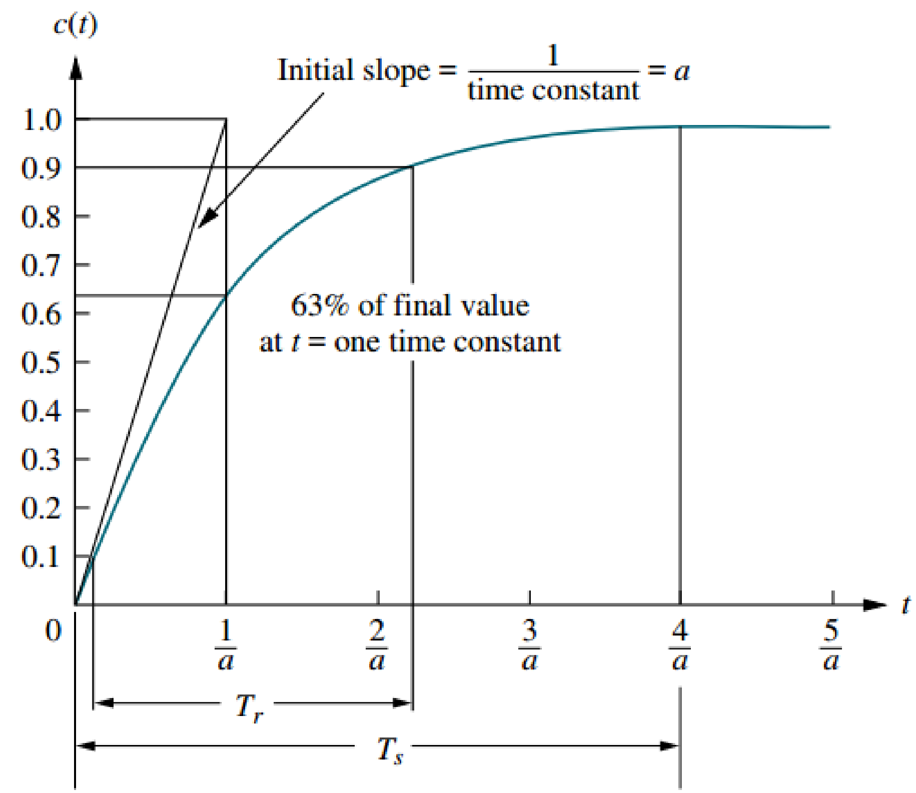
Review : First-order system





$$C(s) = R(s)G(s) = \frac{a}{s(s+a)}$$

$$\boxed{c(t) = c_f(t) + c_n(t) = 1 - e^{-at}}$$

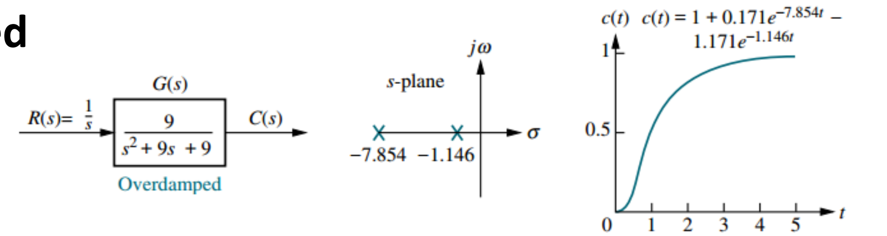


1. Time constant
2. Rise time
3. Settling time

Review : Second-order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

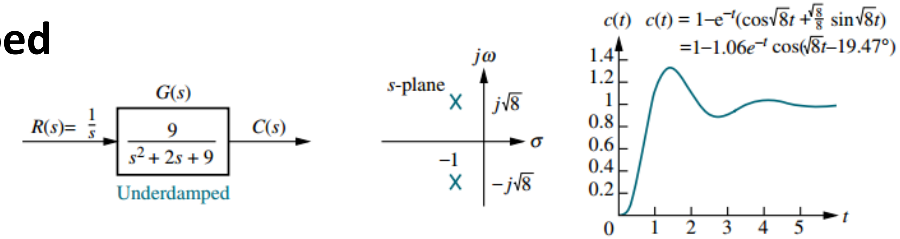
Overdamped



Poles: Two real at $-\sigma_1,\ -\sigma_2$

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

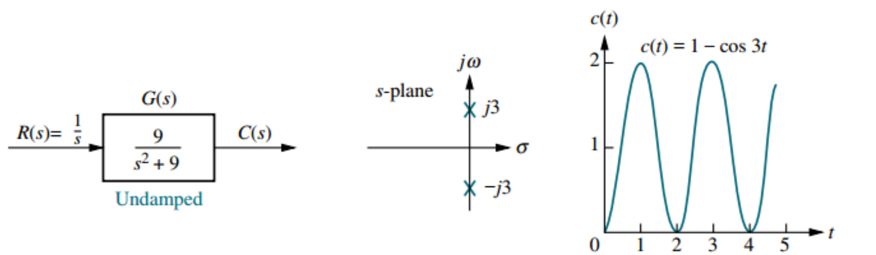
Underdamped



Poles: Two complex at $-\sigma_d \pm j\omega_d$

$$c(t) = Ae^{-\sigma_d t}\cos(\omega_d t - \phi)$$

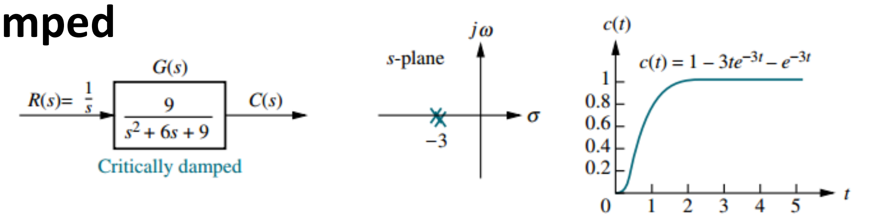
Undamped



Poles: Two imaginary at $\pm j\omega_1$

$$c(t) = A\cos(\omega_1 t - \phi)$$

Critically damped

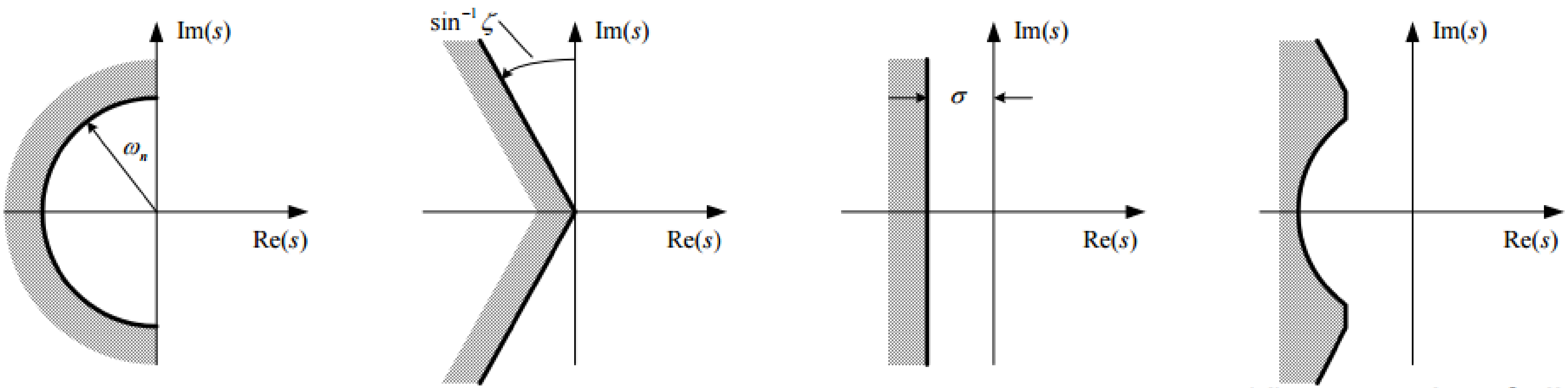


Poles: Two real at $-\sigma_1$

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 t e^{-\sigma_1 t}$$

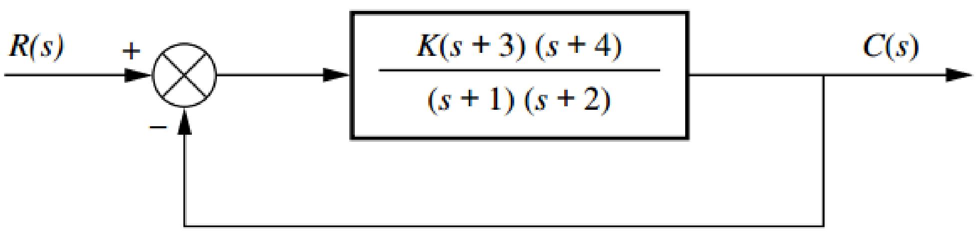
Review : Time domain design specification

1. Peak time: $T_p = \dfrac{\pi}{\omega_d}$
2. Percent overshoot:
$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$
3. Rise time:
$$\omega_n T_r = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1$$
4. Settling time: $T_s = \dfrac{4}{\sigma_d}$



(a) rise time (b) overshoot (c) settling time (d) composite of all three requirements

Review : Root locus techniques



$$|KG(s)H(s)| = 1$$

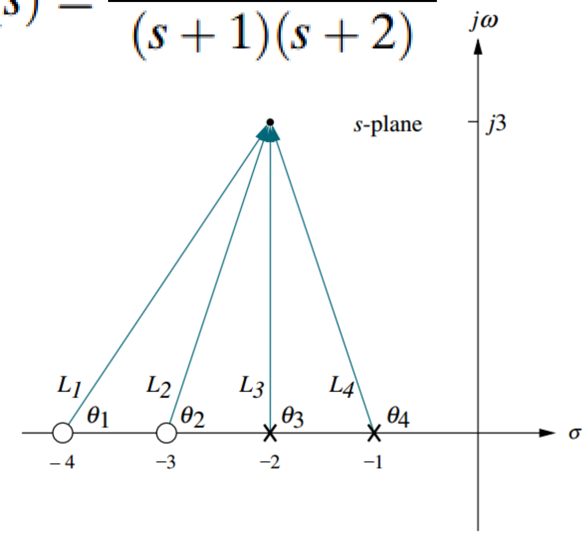
$$\angle KG(s)H(s) = (2k+1)180^\circ$$

Open-loop TF

$$KG(s)H(s) = \frac{K(s+3)(s+4)}{(s+1)(s+2)}$$

Closed-loop TF

$$T(s) = \frac{K(s+3)(s+4)}{(1+K)s^2 + (3+7K)s + (2+12K)}$$



$-2 + j3$ vs $-2 + j(\sqrt{2}/2)$

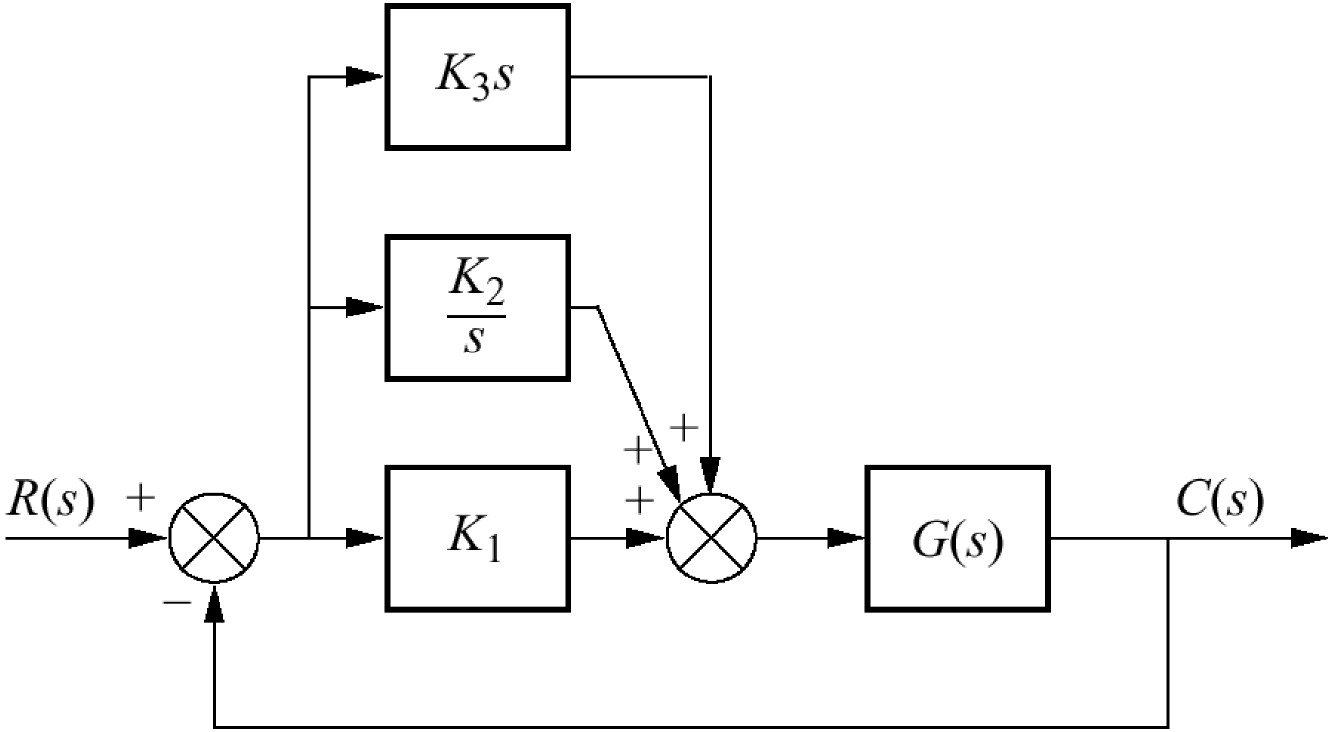
$$M = \frac{\prod \text{zero lengths}}{\prod \text{pole lengths}} = \frac{\prod_{i=1}^{m} |(s+z_i)|}{\prod_{j=1}^{n} |(s+p_j)|}$$

$$\theta = \sum \text{zero angles} - \sum \text{pole angles} = \sum_{i=1}^{m} \angle(s+z_i) - \sum_{j=1}^{n} \angle(s+p_j)$$

Review : Improving transient response

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 + K_2 + K_3 s^2}{s} = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s}$$

PID controller or using passive network it's called lag-lad compensator



Review : Improving transient response

Table 9.7 Types of cascade compensators

Function	Compensator	Transfer function	Characteristics
Improve steady-state error	PI	$K\frac{s+z_c}{s}$	1. Increases system type. 2. Error becomes zero. 3. Zero at $-z_c$ is small and negative. 4. Active circuits are required to implement.
Improve steady-state error	Lag	$K\frac{s+z_c}{s+p_c}$	1. Error is improved but not driven to zero. 2. Pole at $-p_c$ is small and negative. 3. Zero at $-z_c$ is close to, and to the left of, the pole at $-p_c$. 4. Active circuits are not required to implement.
Improve transient response	PD	$K(s+z_c)$	1. Zero at $-z_c$ is selected to put design point on root locus. 2. Active circuits are required to implement. 3. Can cause noise and saturation; implement with rate feedback or with a pole (lead).
Improve transient response	Lead	$K\frac{s+z_c}{s+p_c}$	1. Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus. 2. Pole at $-p_c$ is more negative than zero at $-z_c$. 3. Active circuits are not required to implement.

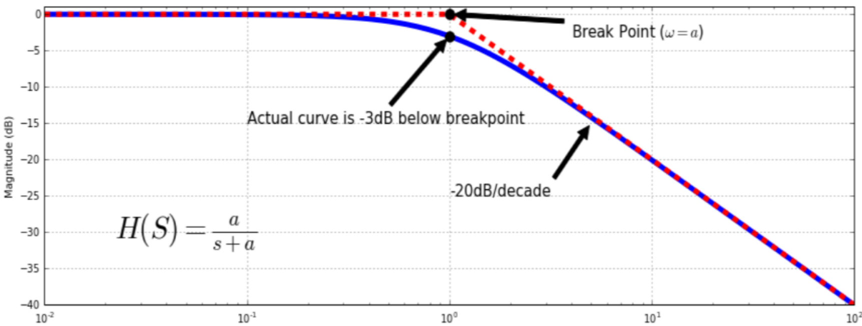
Review : Improving transient response

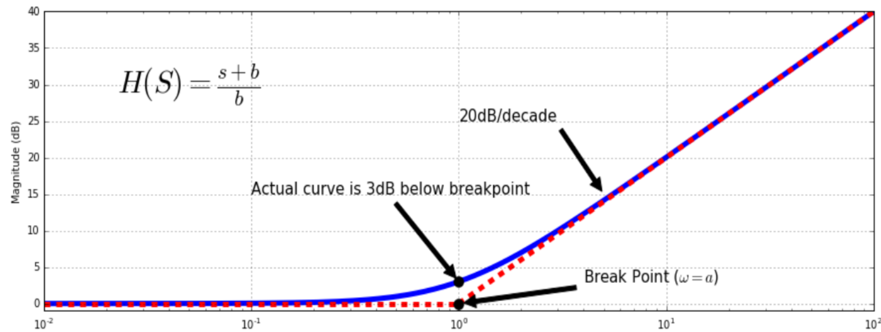
Improve steady-state error and transient response	PID	$K\dfrac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{s}$	1. Lag zero at $-z_{\text{lag}}$ and pole at origin improve steady-state error. 2. Lead zero at $-z_{\text{lead}}$ improves transient response. 3. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, the origin. 4. Lead zero at $-z_{\text{lead}}$ is selected to put design point on root locus. 5. Active circuits required to implement. 6. Can cause noise and saturation; implement with rate feedback or with an additional pole.
Improve steady-state error and transient response	Lag-lead	$K\dfrac{(s+z_{\text{lag}})(s+z_{\text{lead}})}{(s+p_{\text{lag}})(s+p_{\text{lead}})}$	1. Lag pole at $-p_{\text{lag}}$ and lag zero at $-z_{\text{lag}}$ are used to improve steady-state error. 2. Lead pole at $-p_{\text{lead}}$ and lead zero at $-z_{\text{lead}}$ are used to improve transient response. 3. Lag pole at $-p_{\text{lag}}$ is small and negative. 4. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, lag pole at $-p_{\text{lag}}$. 5. Lead zero at $-z_{\text{lead}}$ and lead pole at $-p_{\text{lead}}$ are selected to put design point on root locus. 6. Lead pole at $-p_{\text{lead}}$ is more negative than lead zero at $-z_{\text{lead}}$. 7. Active circuits are not required to implement.

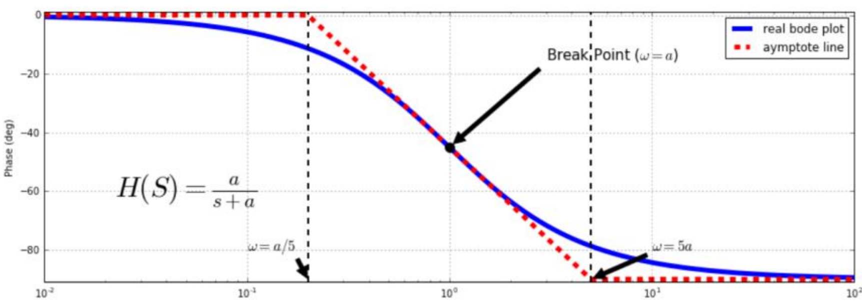
Review : Bode plot

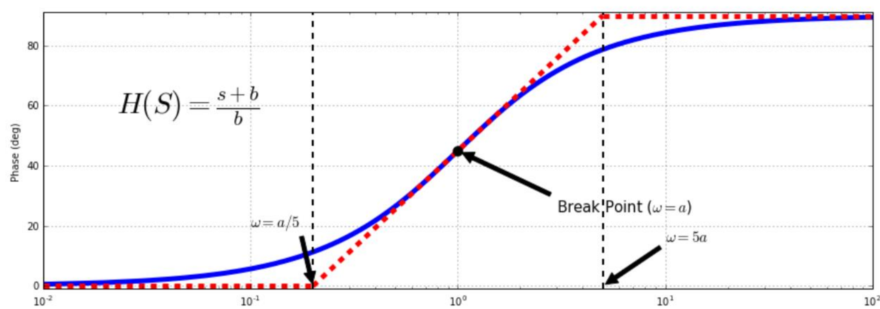
$$H(s) = \frac{a}{s+a}$$

$$H(s) = \frac{s+b}{b}$$



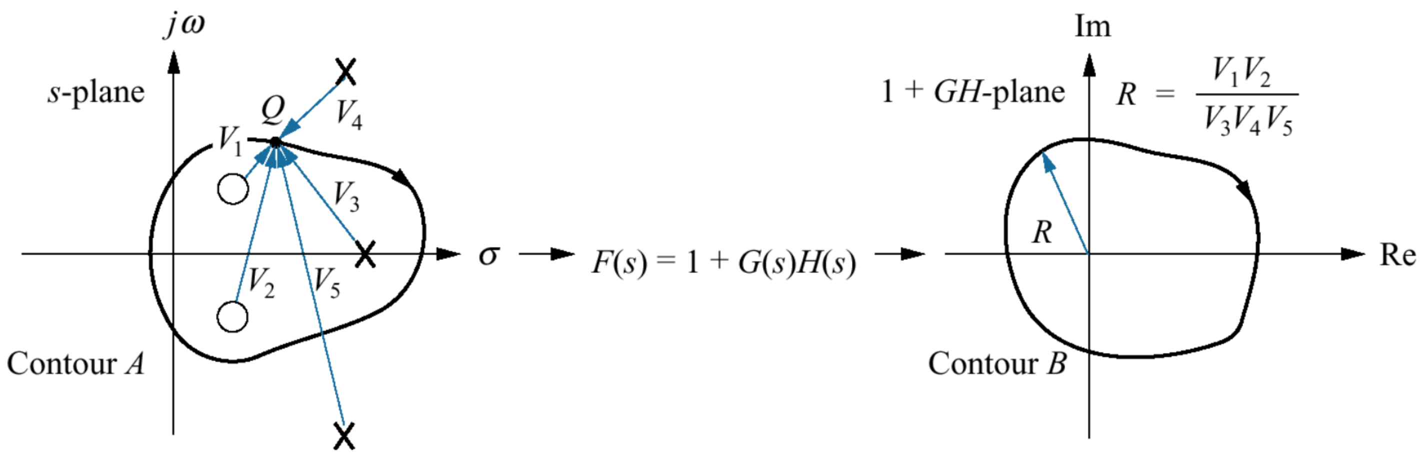






Review : Nyquist plot

❖ Derivation of the Nyquist criterion



Principle of the Argument:

N = P - Z where

N rotations of *F(s)* along Contour B about the origin.

Z zeros of *F(s)* **(closed-loop poles)** inside Contour A.

P poles of *F(s)* **(open-loop poles)** inside Contour A.

In Fig. 10.23, P=1, Z=2 N = 1 - 2 = -1

1 rotation of F(s) along Contour B in the same direction as Contour A

Review : Stability, GM, PM

Two quantitative measures of how stable a system is

- *Gain margin, G_M* – the change in OL gain, expressed in dB, required at 180° of phase shift to make the CL system unstable
- *Phase margin, Φ_M* – the change in OL phase shift required at unity gain to make the CL system unstable

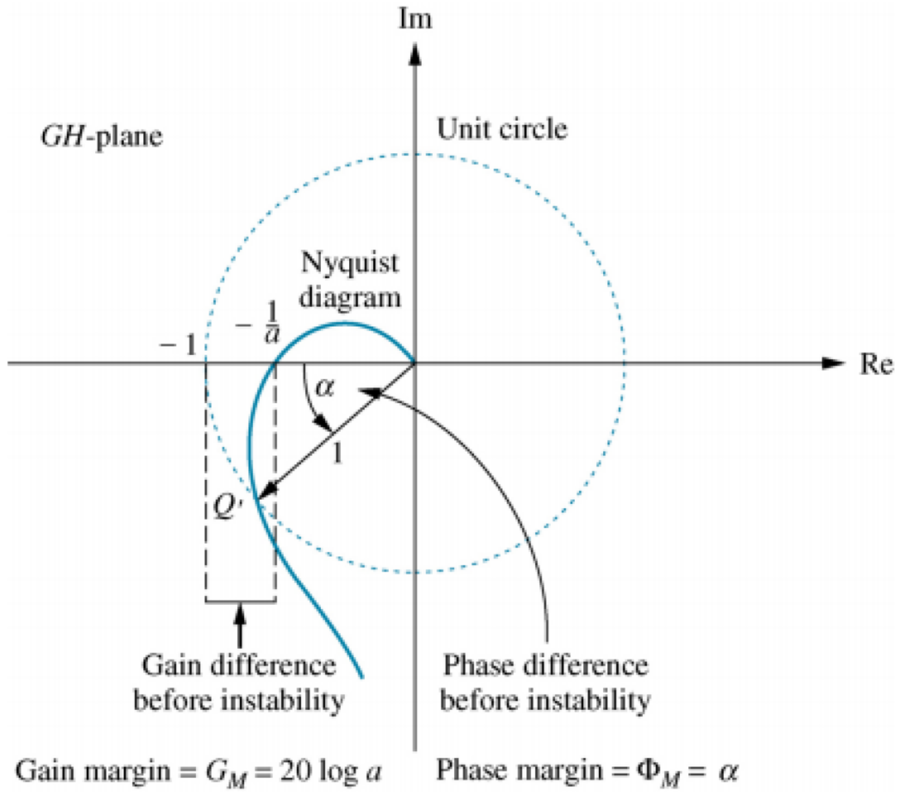
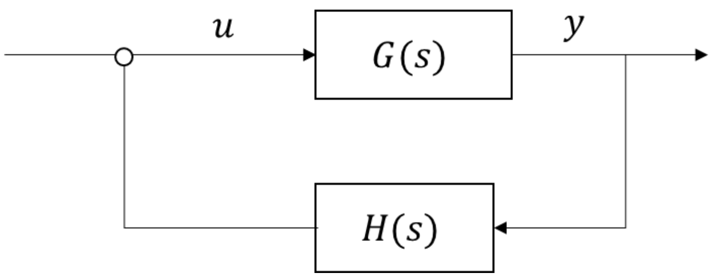
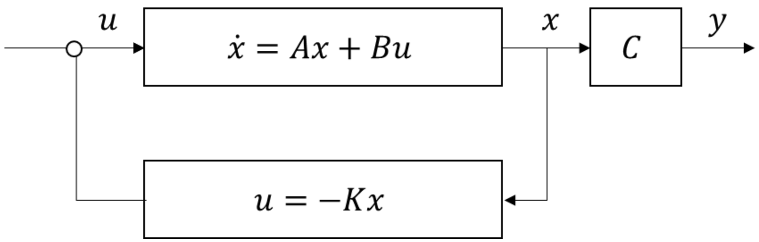


Figure: Nyquist diagram showing gain and phase margins

What is our goal?





Transfer function

$$\frac{1}{ms^2 + bs + k}$$

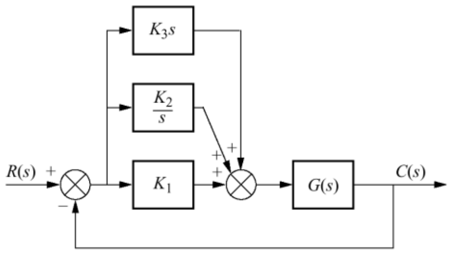
State-space

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu + B_\phi \phi_d$$

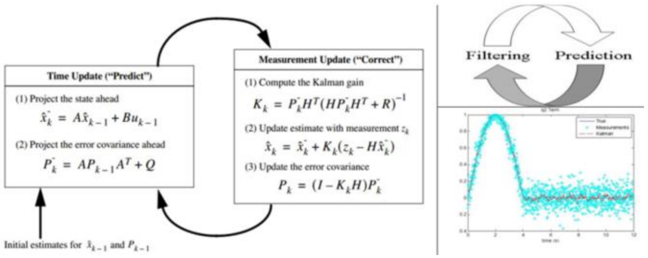
where,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ b_{21} \\ 0 \\ b_{41} \end{bmatrix}, \quad B_\phi = \begin{bmatrix} 0 \\ a_{24} - V_x \\ 0 \\ a_{44} \end{bmatrix}$$

PID control



State feedback, LQ, Kalman filter…



Q & A

수업관련

- 출석 : 게시판 과제 제출 / 전자출결

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