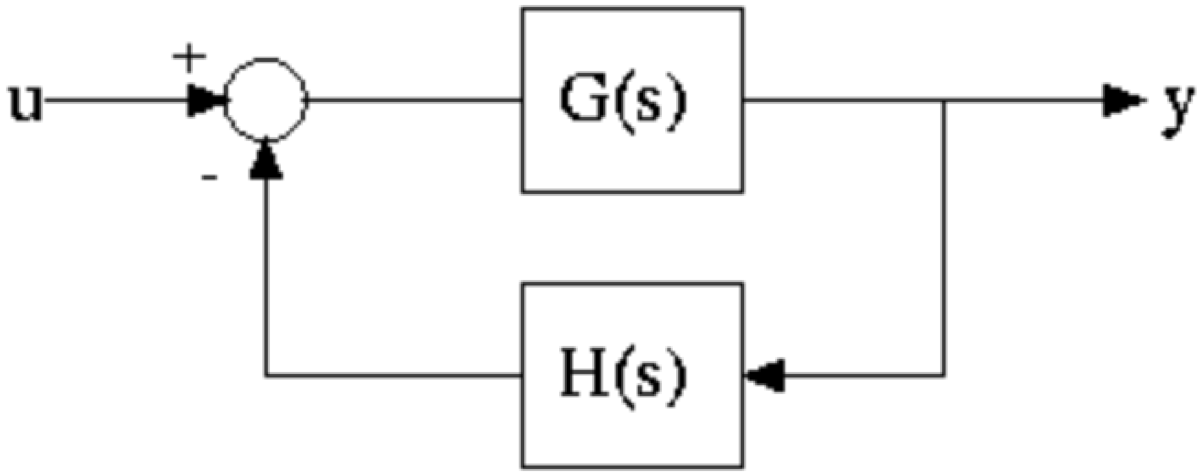
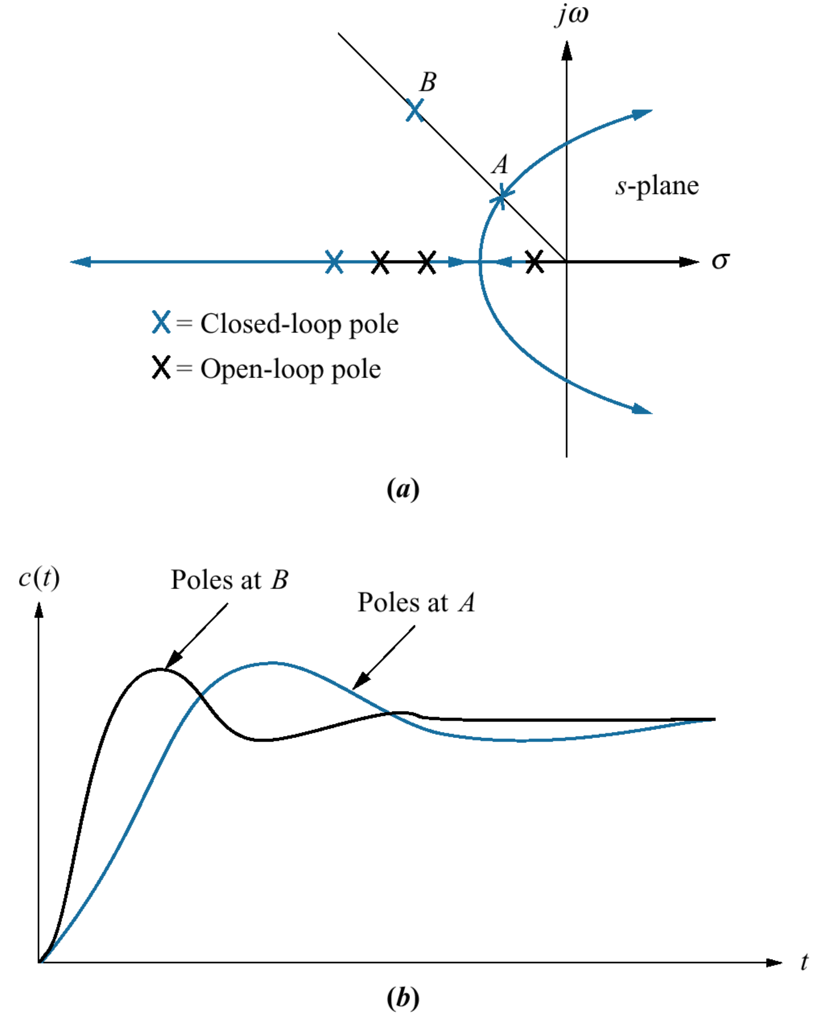
System Control

Design via root locus



Improving transient response

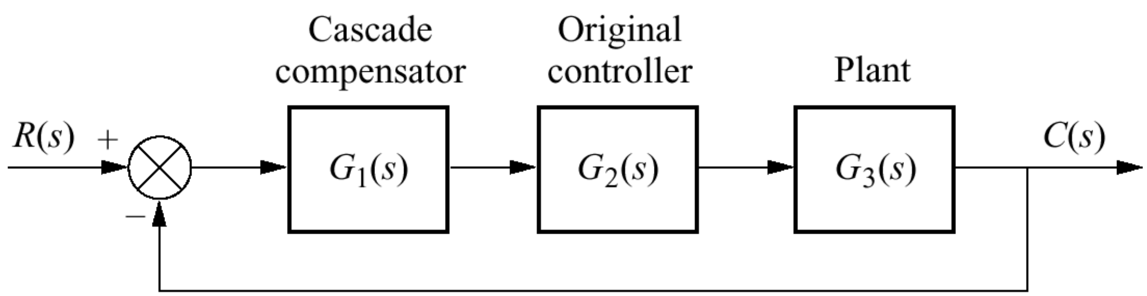
Figure 9.1
a. *Sample root locus, showing possible design point via gain adjustment (A) and desired design point that cannot be met via simple gain adjustment (B);*
b. *responses from poles at A and B*



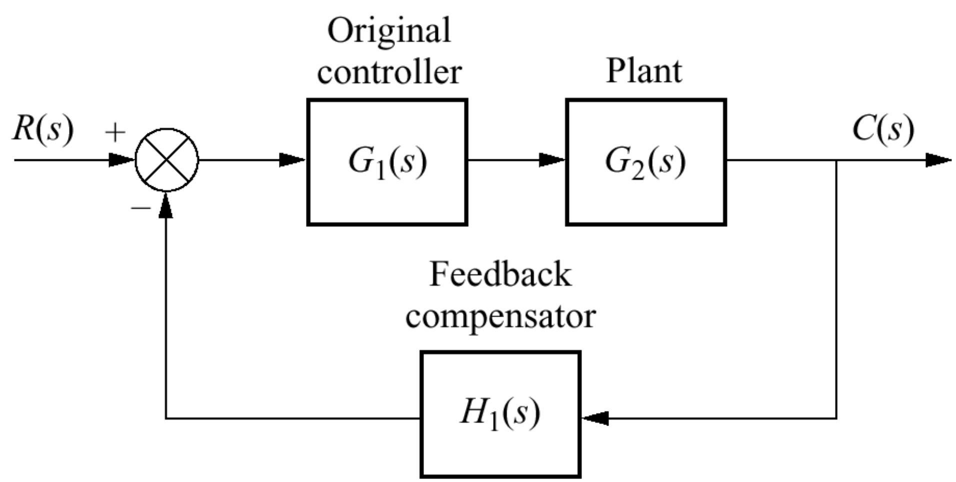
Improving steady-state error

Compensation techniques:
a. *cascade;*
b. *feedback*

Ideal compensators are implemented with active networks.



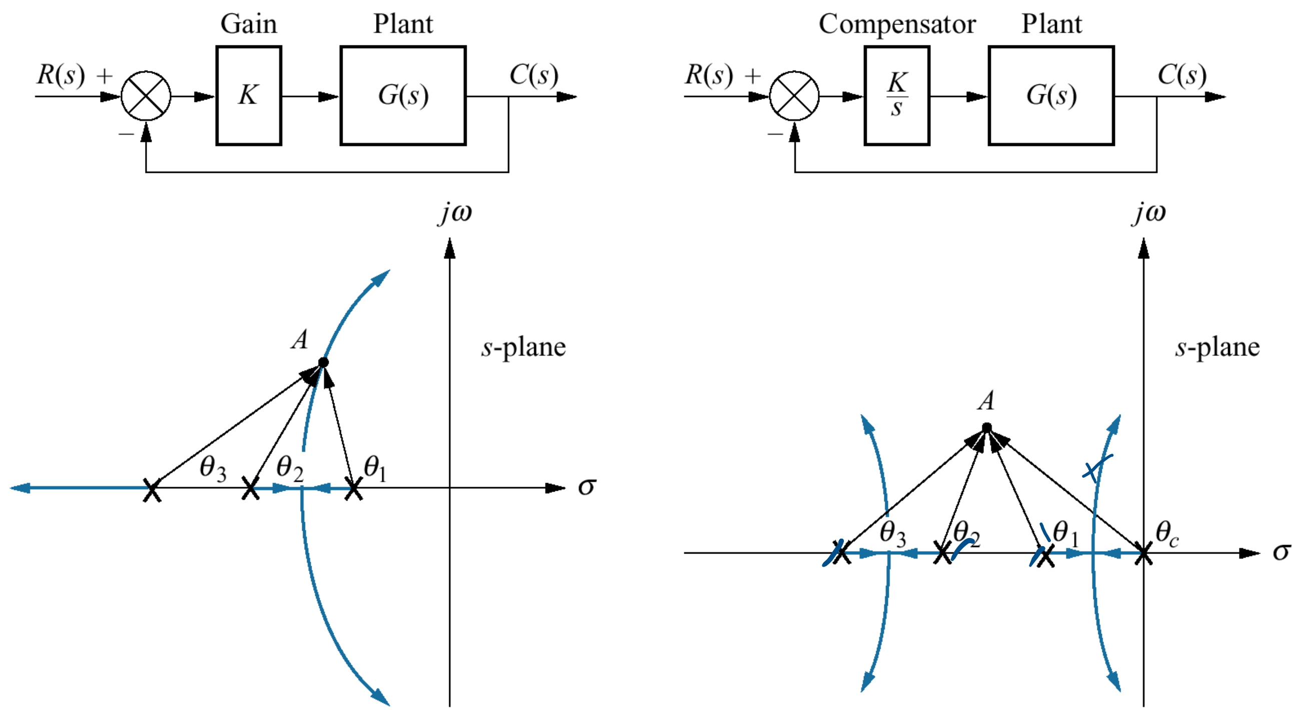
(*a*)



(*b*)

Improving steady-state error via cascade compensation

Pole at A is:
a. *on the root locus without compensator;*
b. *not on the root locus with compensator pole added;*
(figure continues)



$-\theta_1 - \theta_2 - \theta_3 = (2k+1)180°$

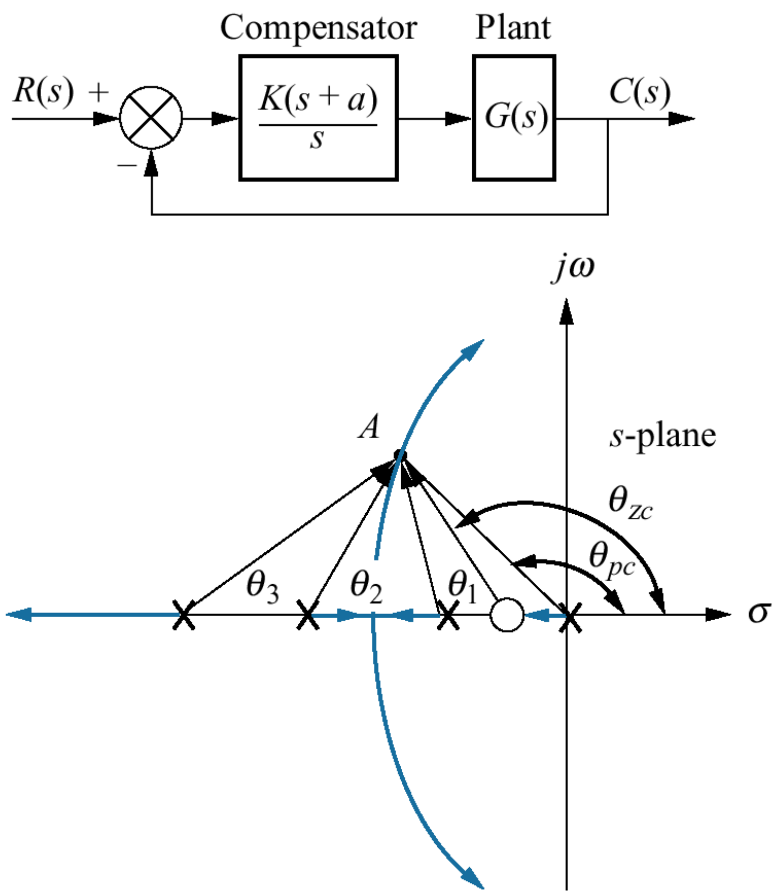
(*a*)

$-\theta_1 - \theta_2 - \theta_3 - \theta_c \neq (2k+1)180°$

(*b*)

Ideal integral compensation (PI)

***c.** approximately on the root locus with compensator pole and zero added*



$$-\theta_1 - \theta_2 - \theta_3 - \theta_{pc} + \theta_{zc} \cong (2k + 1)180°$$

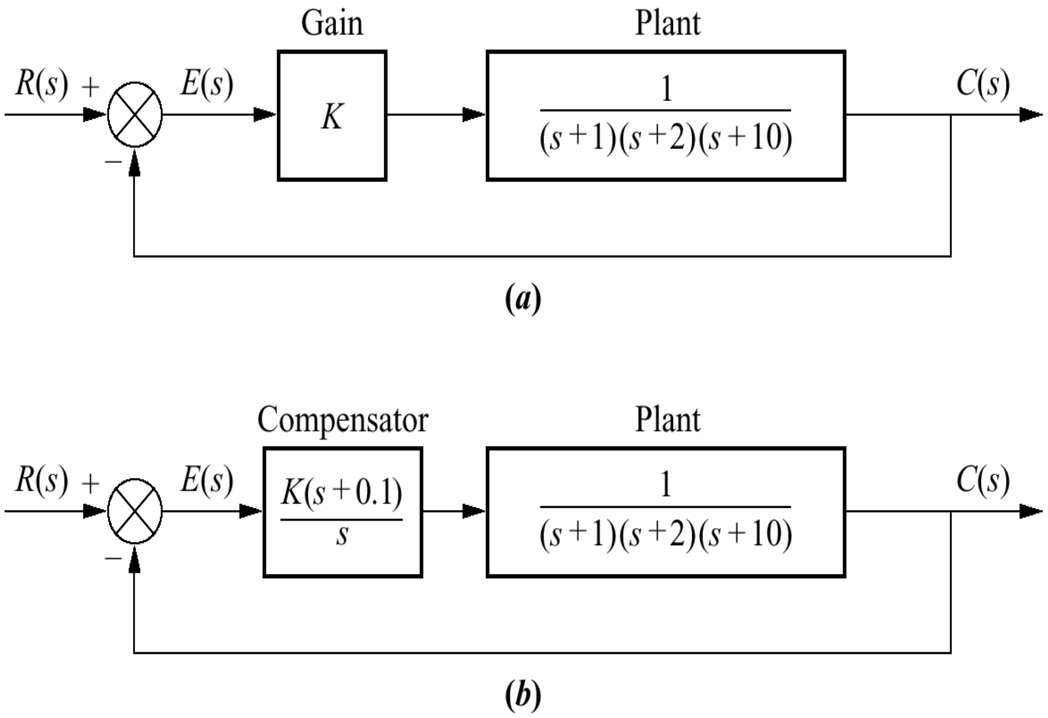
(c)

Closed-loop system for example 9.1

Closed-loop system for Example 9.1

a. *before compensation;*
b. *after ideal integral compensation*

Problem: The given system operating with damping ratio of 0.174. Add an ideal integral compensator to reduce the ss error.

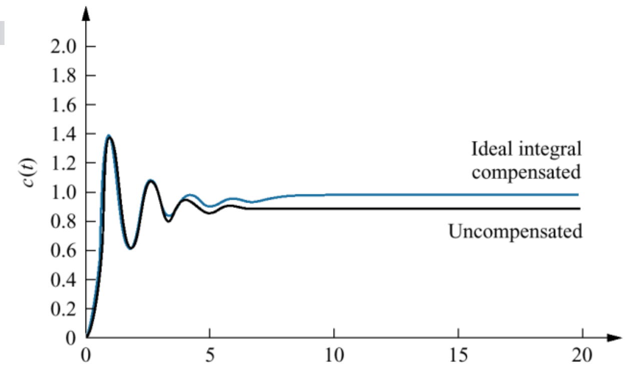


Solution:

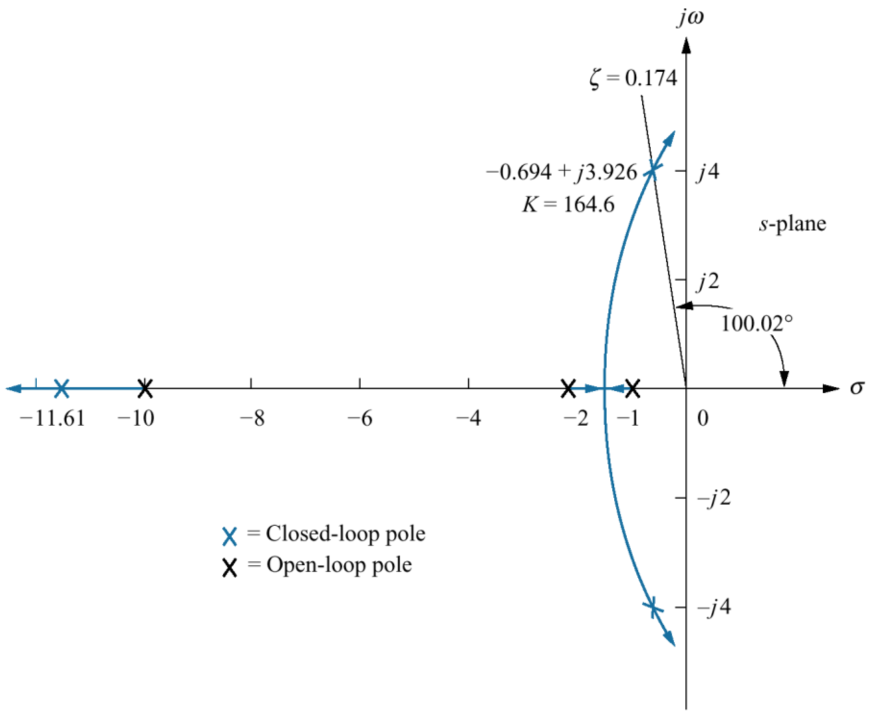
We compensate the system by choosing a pole at the origin and a zero at -0.1

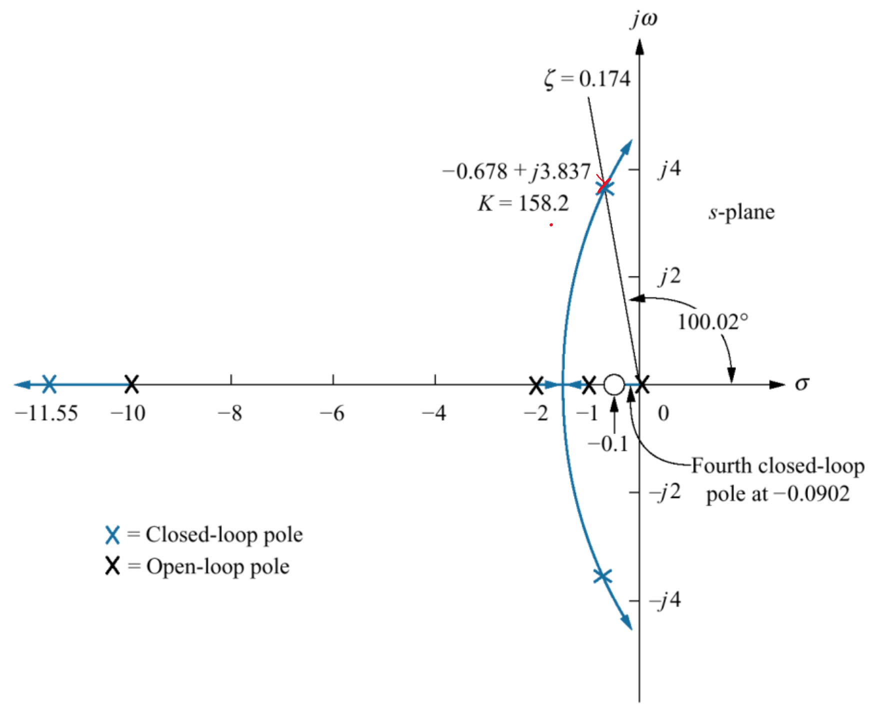
Root locus for compensated system

The gain K = 164.6 yields K_p = 8.23 and $e(\infty) = \dfrac{1}{1+K_p} = 0.108$

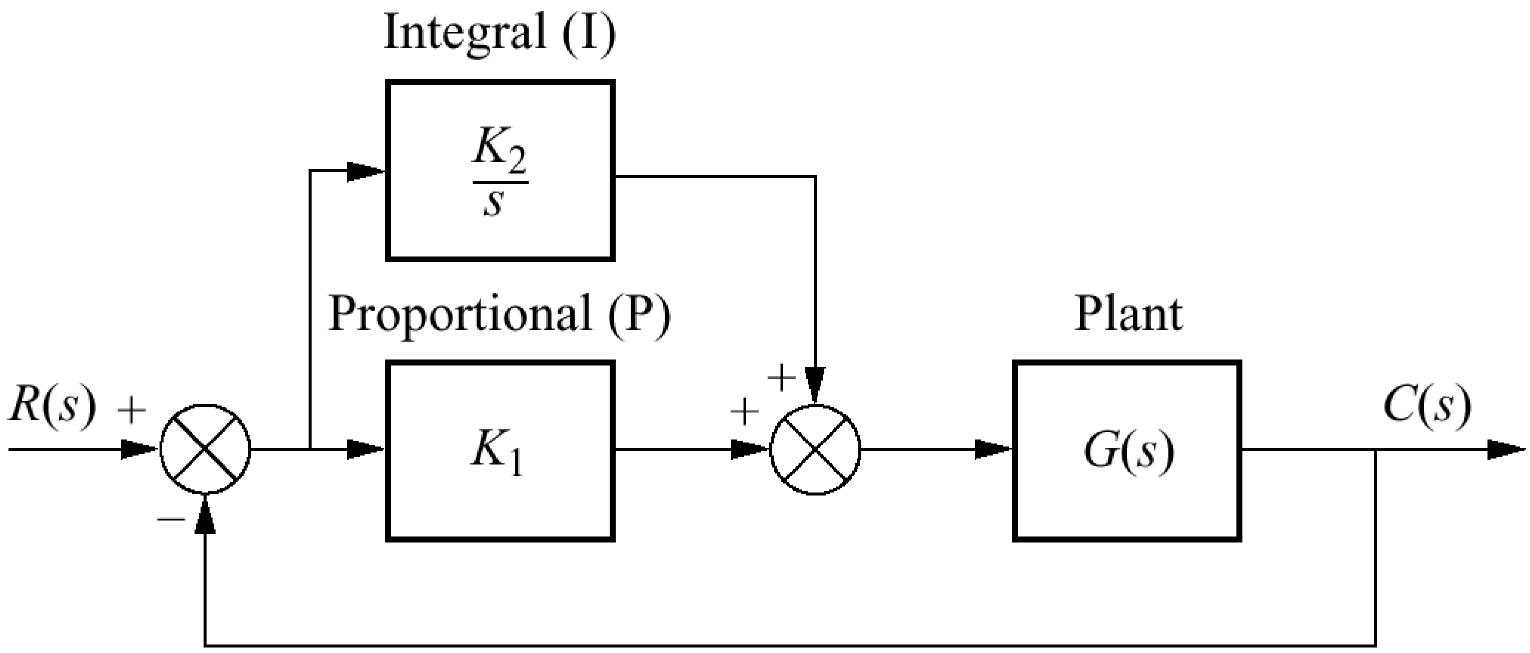


Almost same transient response and gain, but with zero ss error since we have a type one system.





PI controller

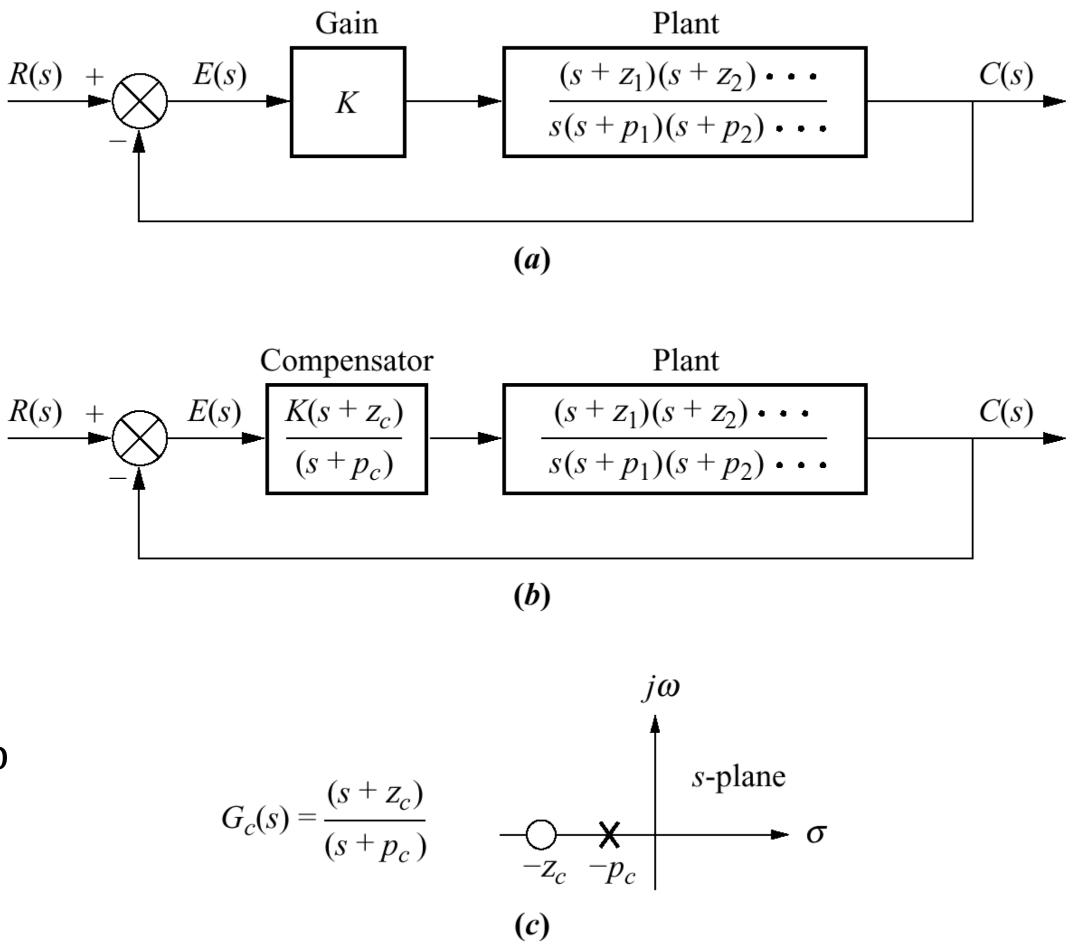


A method to implement an Ideal integral compensator is shown.

$$G_c(s) = K_1 + \frac{K_2}{s} = \frac{K_1\left(s + \frac{K_2}{K_1}\right)}{s}$$

Lag compensator

a. *Type 1 uncompensated system;*
b. *Type 1 compensated system;*
c. *compensator pole-zero plot*



Using passive networks, the compensation pole and zero is moved to the left, close to the origin.

The static error constant for uncompensated system is

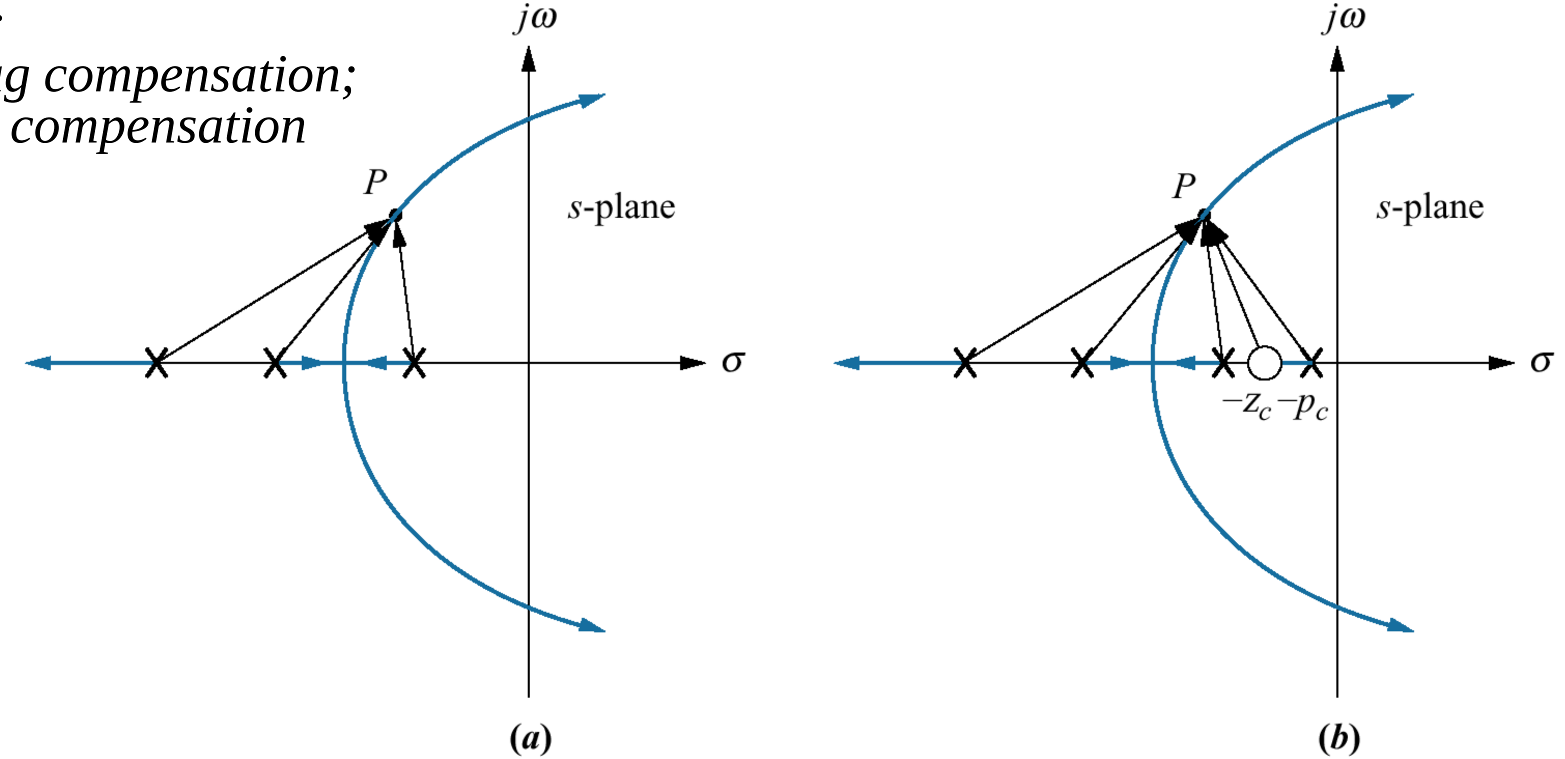
$$K_{vo} = \frac{Kz_1 z_2 ....}{p_1 p_2 ....}$$

Assuming the compensator is used as in b & c the static error is

$$K_{vN} = \frac{(Kz_1 z_2 ....)(z_c)}{(p_1 p_2 ....)(p_c)}$$

Effect on transient response

Root locus:
***a.** before lag compensation;*
***b.** after lag compensation*

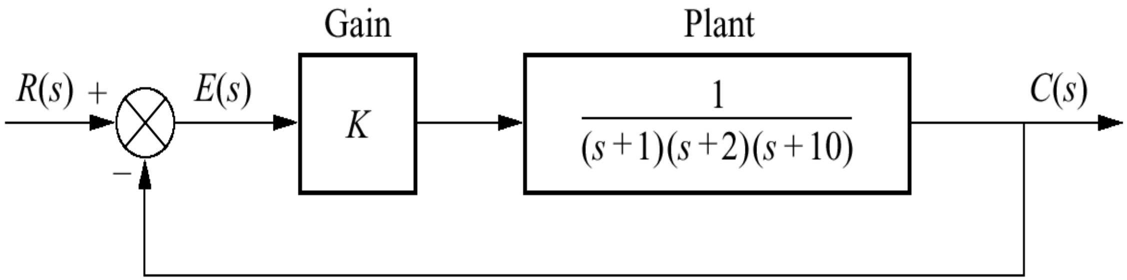


Almost no change on the transient response and same gain K. While the ss error is effected since

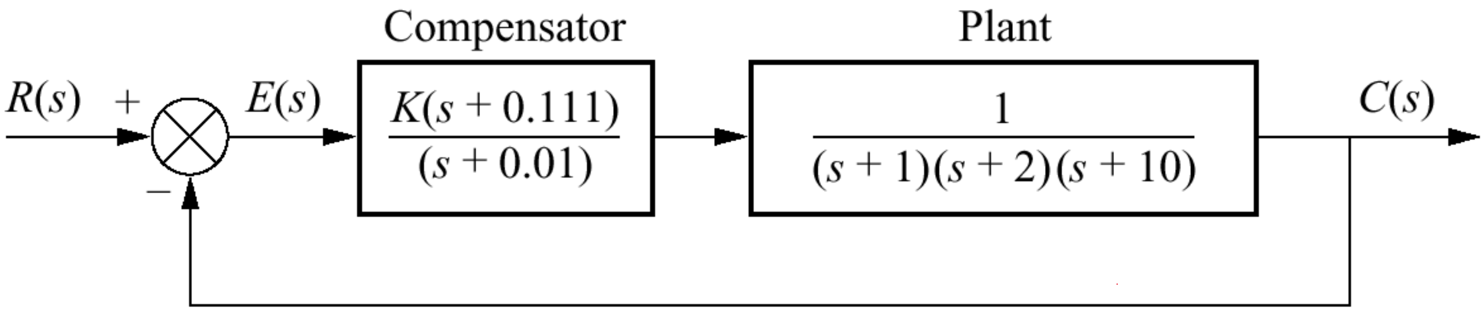
$$K_{vN} = K_{vo}\frac{z_c}{p_c} > K_{vo}$$

Lag compensator design example 9.2

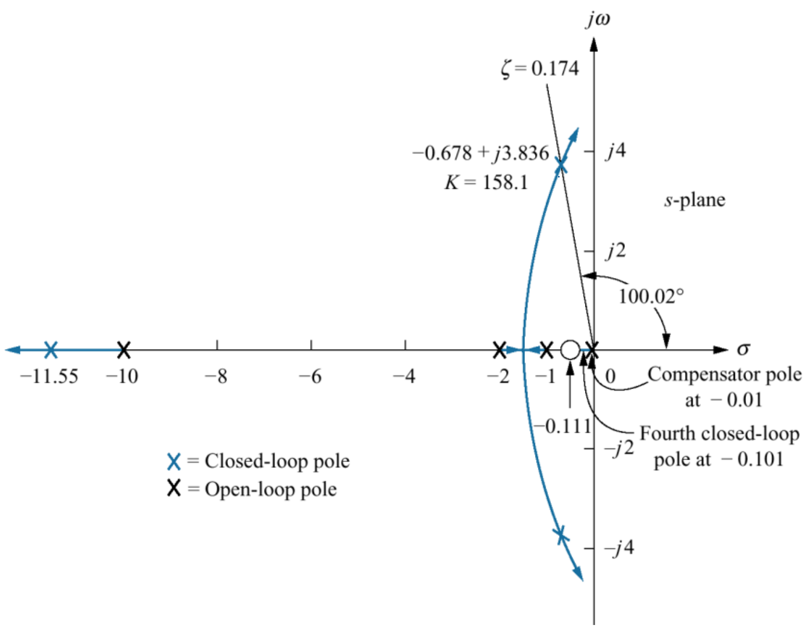
Problem: Compensate the shown system to improve the ss error by a factor of 10 if the system is operating with a damping ratio of 0.174



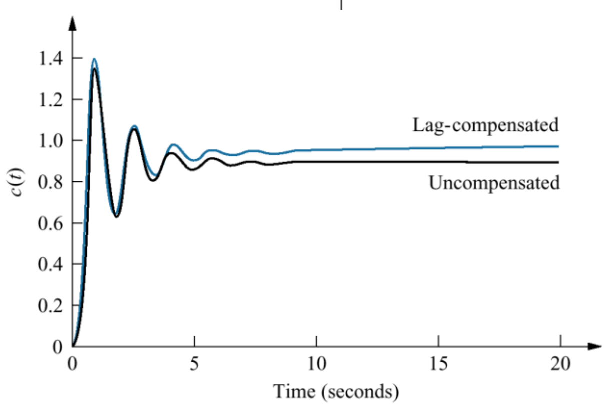
Solution: the uncompensated system error from previous example is 0.108 with $K_p = 8.23$. a ten fold improvement means ss error = 0.0108 so $Kp = 91.59$. so the ratio $\frac{z_c}{p_c} = \frac{K_{PN}}{K_{Po}} = \frac{91.59}{8.23} = 11.13$ arbitrarily selecting $P_c = 0.01$ and $Z_c = 11.13 P_c \approx 0.111$

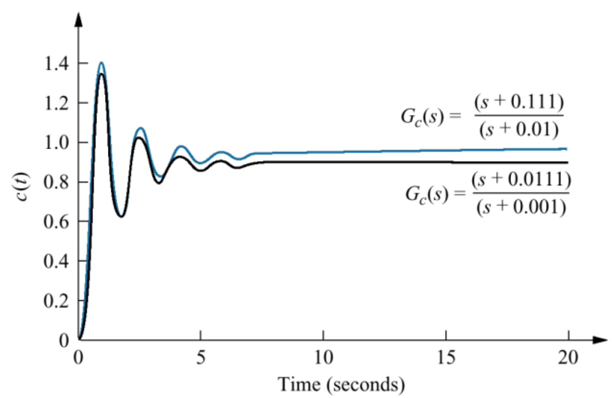


Root locus for compensated system



Parameter	Uncompensated	Lag-compensated
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+10)}$	$\frac{K(s+0.111)}{(s+1)(s+2)(s+10)(s+0.01)}$
K	164.6	158.1
K_p	8.23	87.75
$e(\infty)$	0.108	0.011
Dominant second-order poles	$-0.694 \pm j3.926$	$-0.678 \pm j3.836$
Third pole	-11.61	-11.55
Fourth pole	None	-0.101
Zero	None	-0.111





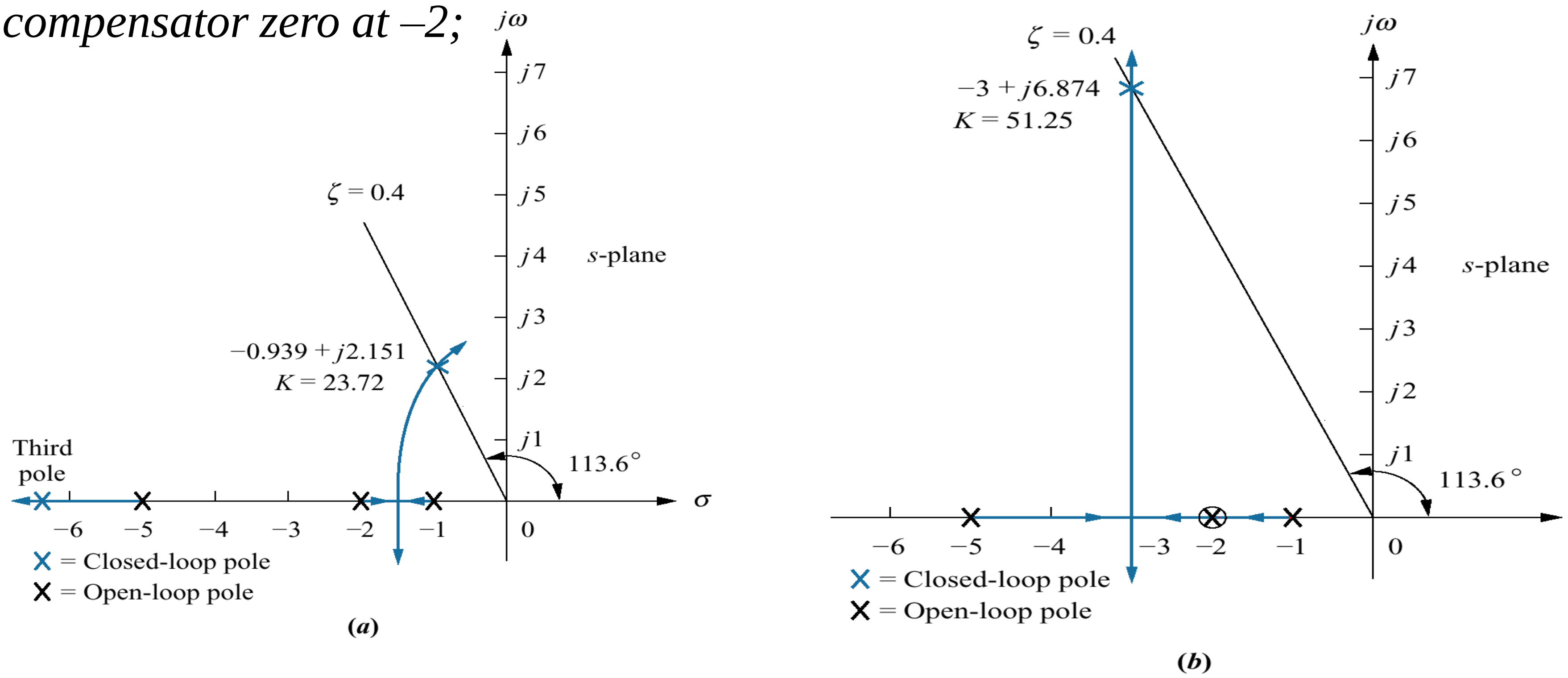
Improving transient response via cascade compensation

Ideal Derivative compensator is called PD controller
When using passive network it's called lead compensator

Using ideal derivative compensation: $G_c(s) = s + z_c$

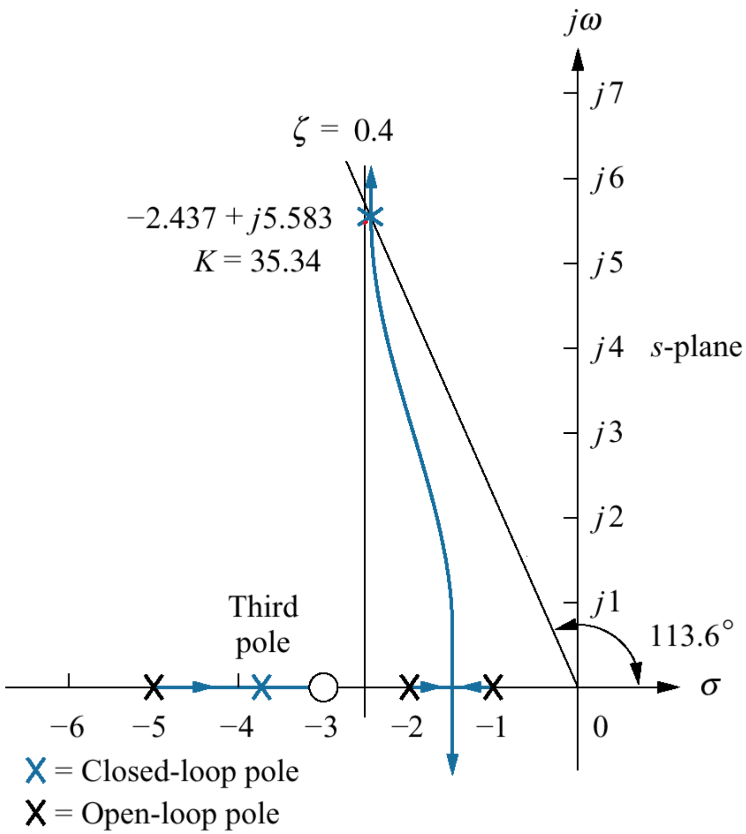
a. *uncompensated;*

b. *compensator zero at –2;*

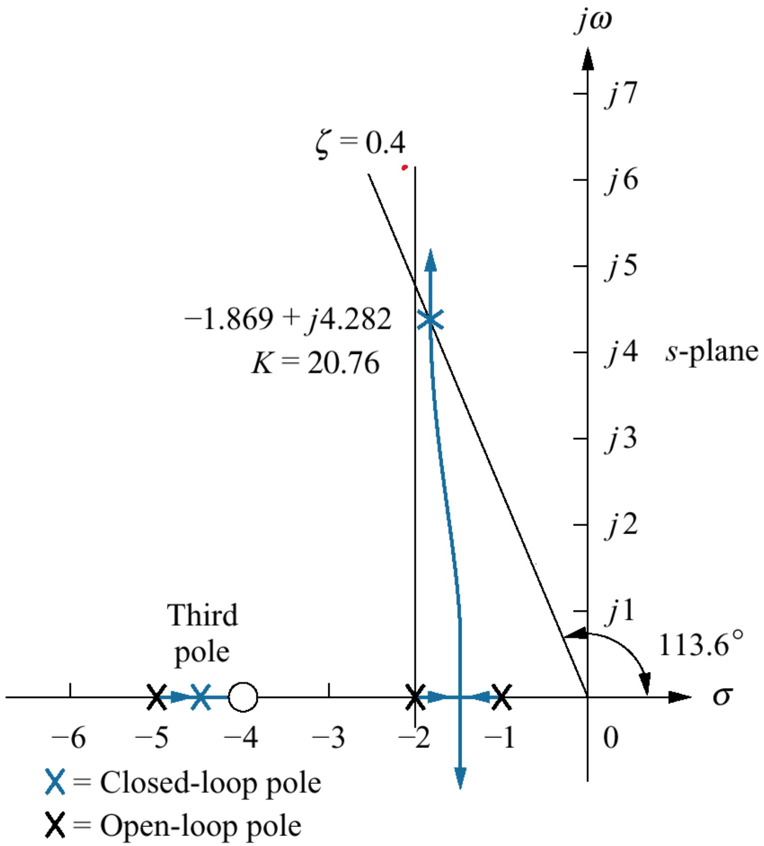


Improving transient response via cascade compensation

***c.** Compensator zero at –3;*
***d.** Compensator zero at – 4*

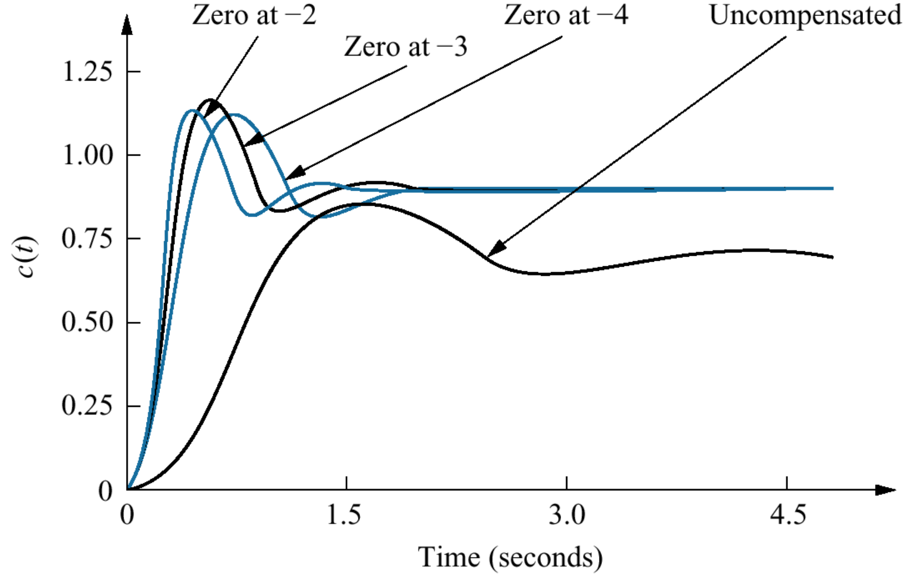


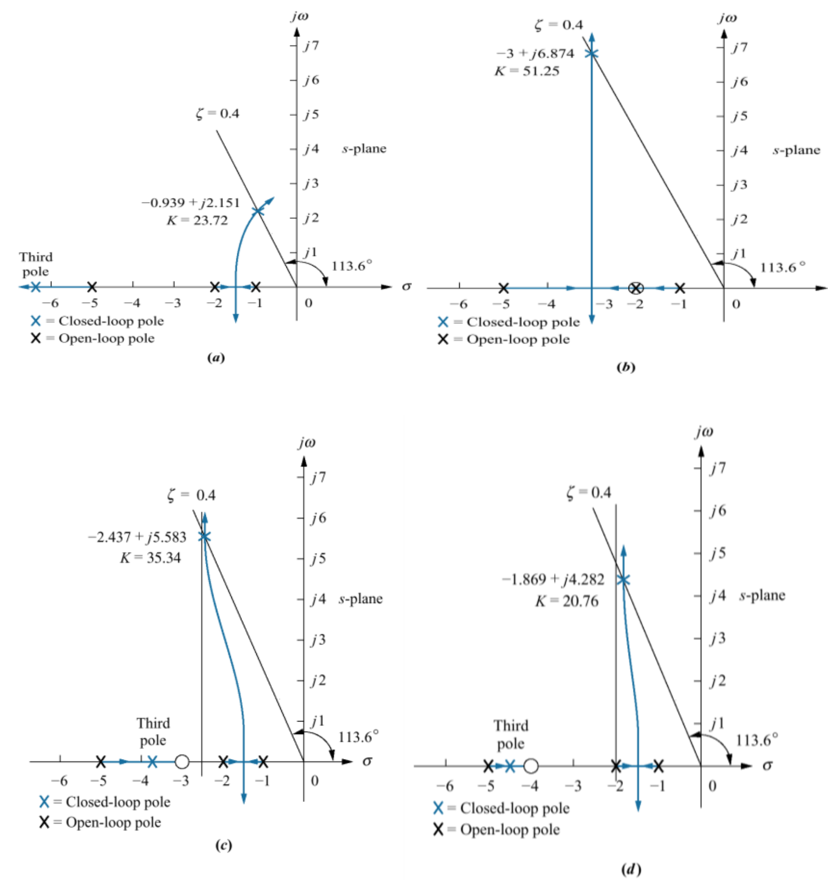
(c)



(d)

Uncompensated system and ideal derivative compensation solutions

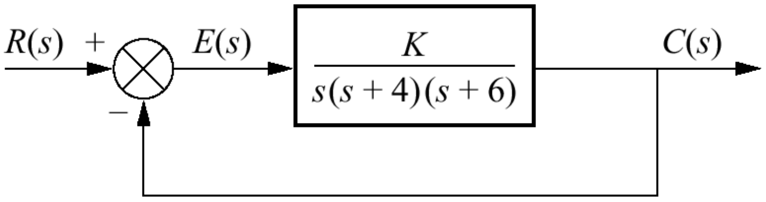




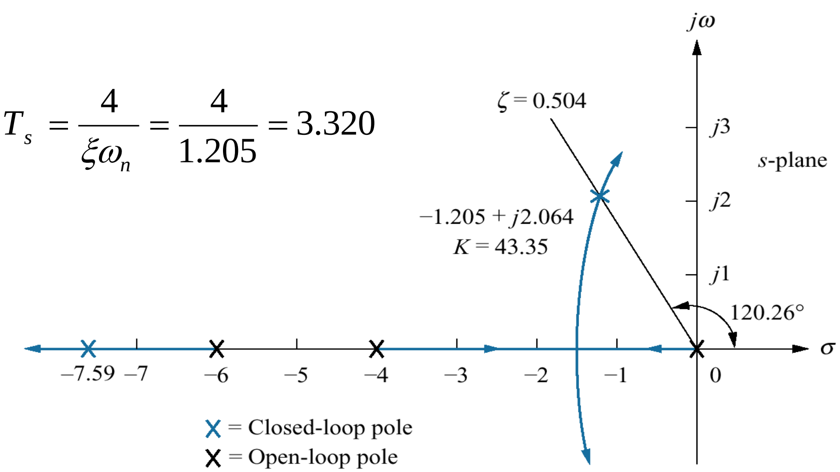
	Uncompensated	**Compensation b**	**Compensation c**	**Compensation d**
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+5)}$	$\frac{K(s+2)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+3)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+4)}{(s+1)(s+2)(s+5)}$
Dom. poles	$-0.939 \pm j2.151$	$-3 \pm j6.874$	$-2.437 \pm j5.583$	$-1.869 \pm j4.282$
K	23.72	51.25	35.34	20.76
ζ	0.4	0.4	0.4	0.4
ω_n	2.347	7.5	6.091	4.673
$\%OS$	25.38	25.38	25.38	25.38
T_s	4.26	1.33	1.64	2.14
T_p	1.46	0.46	0.56	0.733
K_p	2.372	10.25	10.6	8.304
$e(\infty)$	0.297	0.089	0.086	0.107
Third pole	−6.123	None	−3.127	−4.262
Zero	None	None	−3	−4
Comments	Second-order approx. OK	Pure second-order	Second-order approx. OK	Second-order approx. OK

Feedback control system for example 9.3

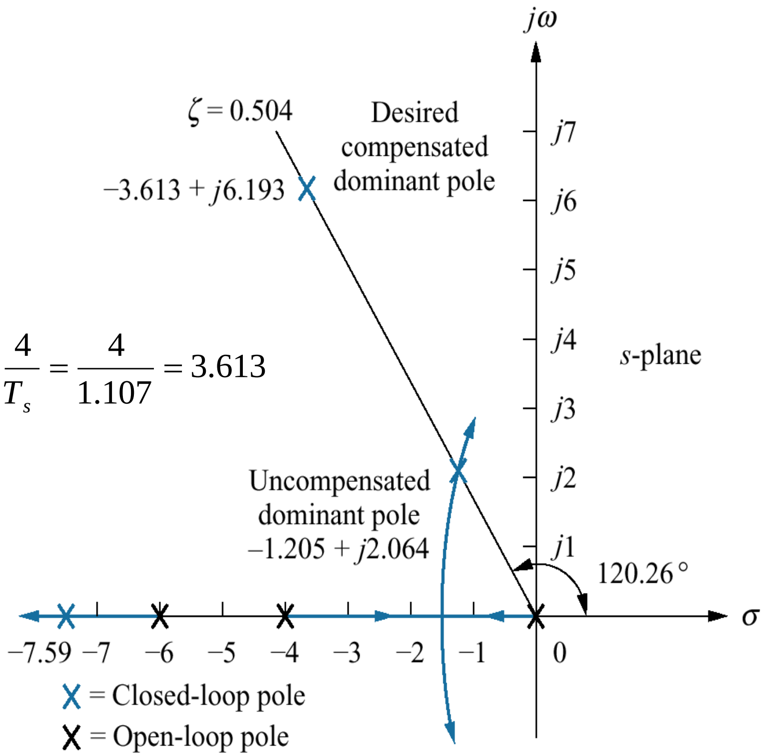
Problem: Given the system in the figure, design an ideal derivative compensator to yield a 16% overshoot with a threefold reduction in settling time.



$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{1.205} = 3.320$$



$$\sigma = \frac{4}{T_s} = \frac{4}{1.107} = 3.613$$



$$\omega_d = 3.613\tan(180^\circ - 120.26^\circ) = 6.193$$

Feedback control system for example 9.3

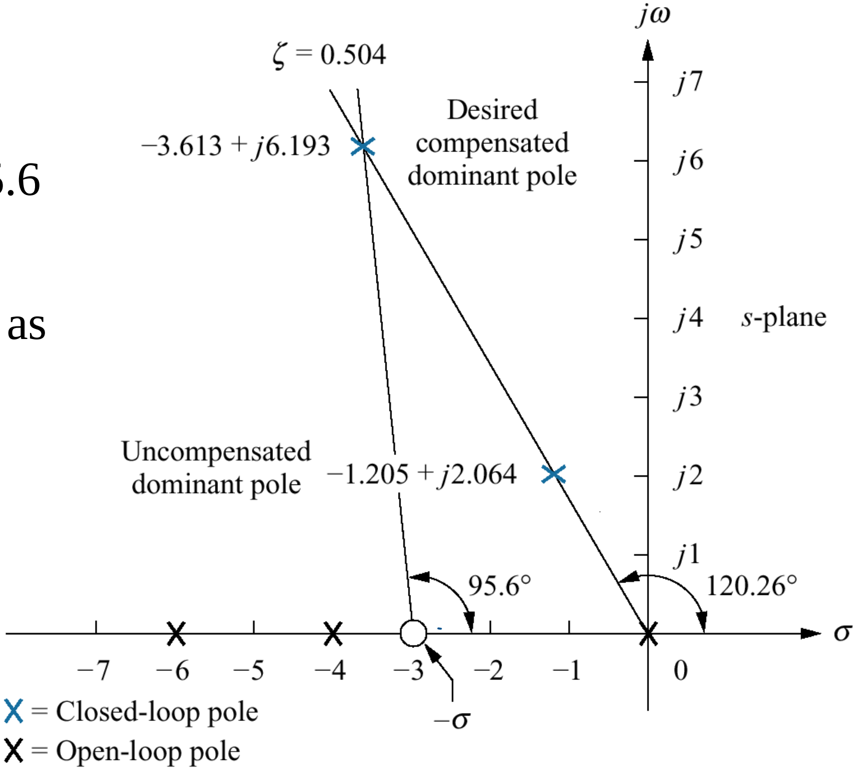
The sum of angles from all poles to the desired compensated pole -3.613+j6.193 is -275.6

The angle of the zero to be on the root locus is 275.6 -180=95.6

The location of the compensator zero is calculated as

$$\frac{6.193}{3.613-\sigma} = \tan(180^\circ - 95.6^\circ)$$

Thus $\sigma = 3.006$



	Uncompensated	Simulation	Compensated	Simulation
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$		$\frac{K(s+3.006)}{s(s+4)(s+6)}$	
Dominant poles	$-1.205 \pm j2.064$		$-3.613 \pm j6.193$	
K	43.35		47.45	
ζ	0.504		0.504	
ω_n	2.39		7.17	
$\%OS$	16	14.8	16	11.8
T_s	3.320	3.6	1.107	1.2
T_p	1.522	1.7	0.507	0.5
K_v	1.806		5.94	
$e(\infty)$	0.554		0.168	
Third pole	−7.591		−2.775	
Zero	None		−3.006	
Comments	Second-order approx. OK		Pole-zero not canceling	

Feedback control system for example 9.3