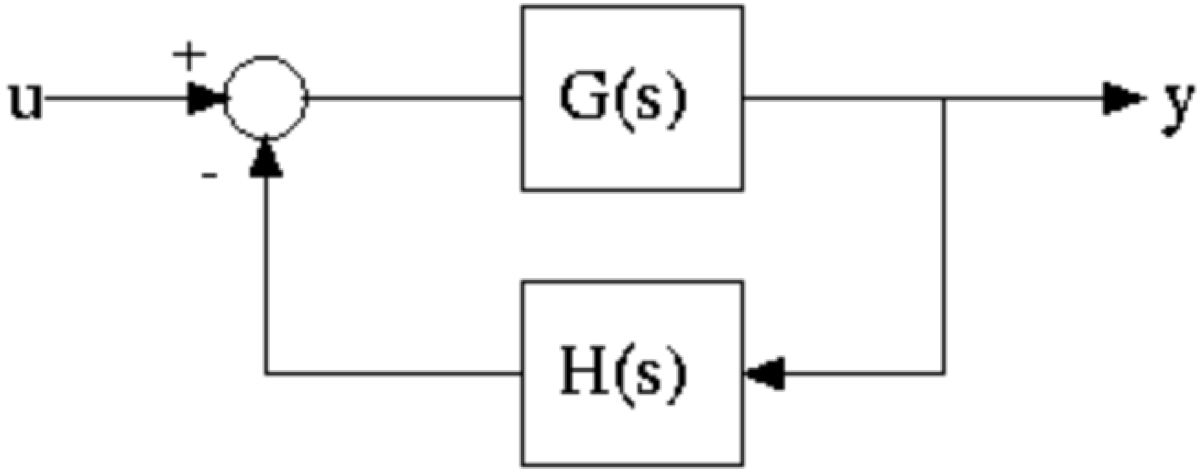
System Control

modeling in the frequency domain



UNICON LAB, Unmanned & Intelligent systems Control Laboratory

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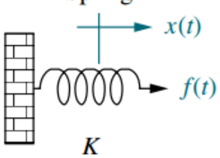
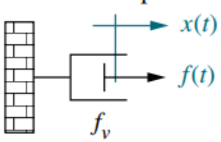
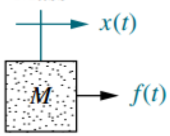
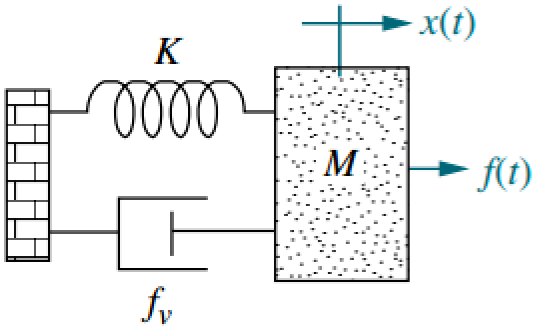
Mechanical system (1)

- Translational mechanical system

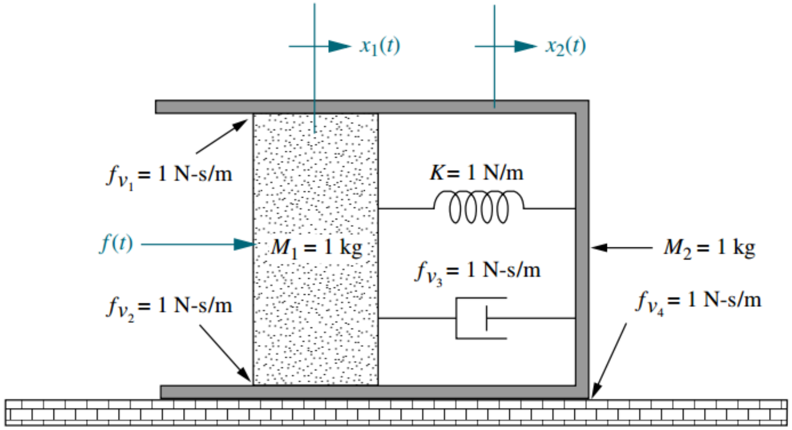
Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m(newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).



$$(Ms^2 + f_v s + K)X(s) = F(s)$$

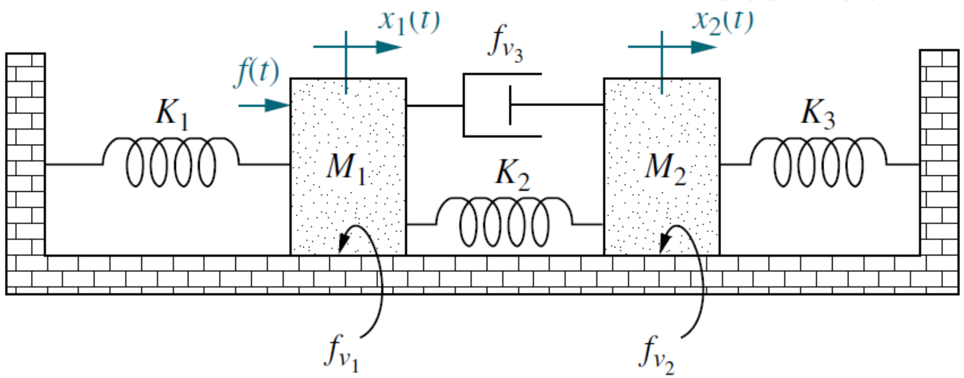
Mechanical system (2)



- M_1 move M_2 hold
- M_1 hold M_2 move
- M_2 move M_1 hold
- M_2 hold M_1 move

Mechanical system (3)

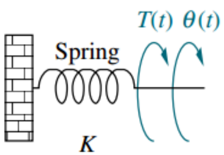
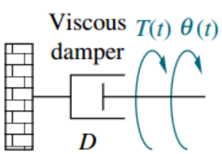
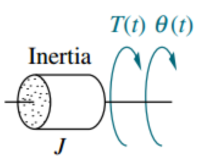
Example Find the transfer function, $X_2(s)/F(s)$



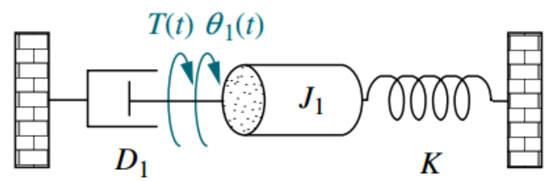
- M_1 move M_2 hold
- M_1 hold M_2 move
- M_2 move M_1 hold
- M_2 hold M_1 move

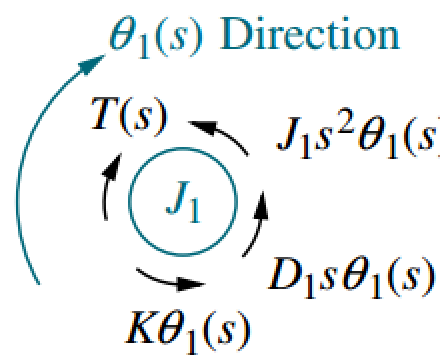
Mechanical system (4)

- Rotational mechanical system

Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$	Ds
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$	Js^2

Note: The following set of symbols and units is used throughout this book: $T(t)$ – N-m (newton-meters), $\theta(t)$ – rad(radians), $\omega(t)$ – rad/s(radians/second), K – N-m/rad(newton- meters/radian), D – N-m-s/rad (newton- meters-seconds/radian). J – kg-m^2(kilograms-meters^2 – newton-meters-seconds^2/radian).





$$(J_1 s^2 + D_1 s + K)\theta_1(s) = T(s)$$

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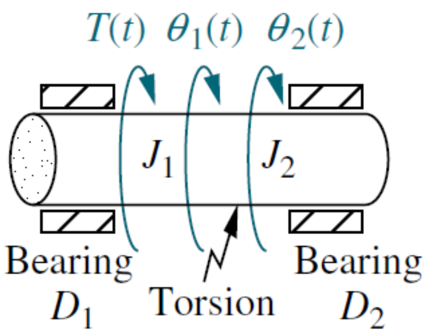
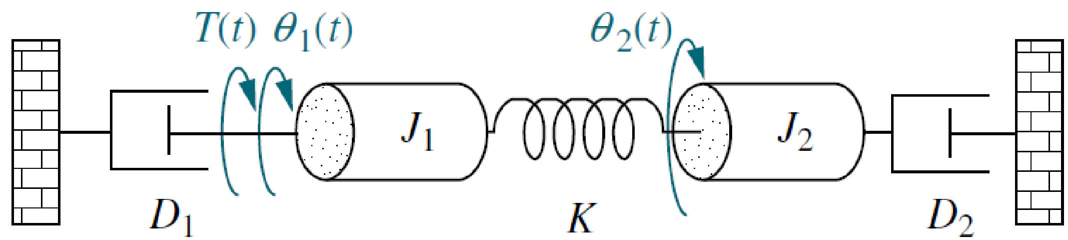
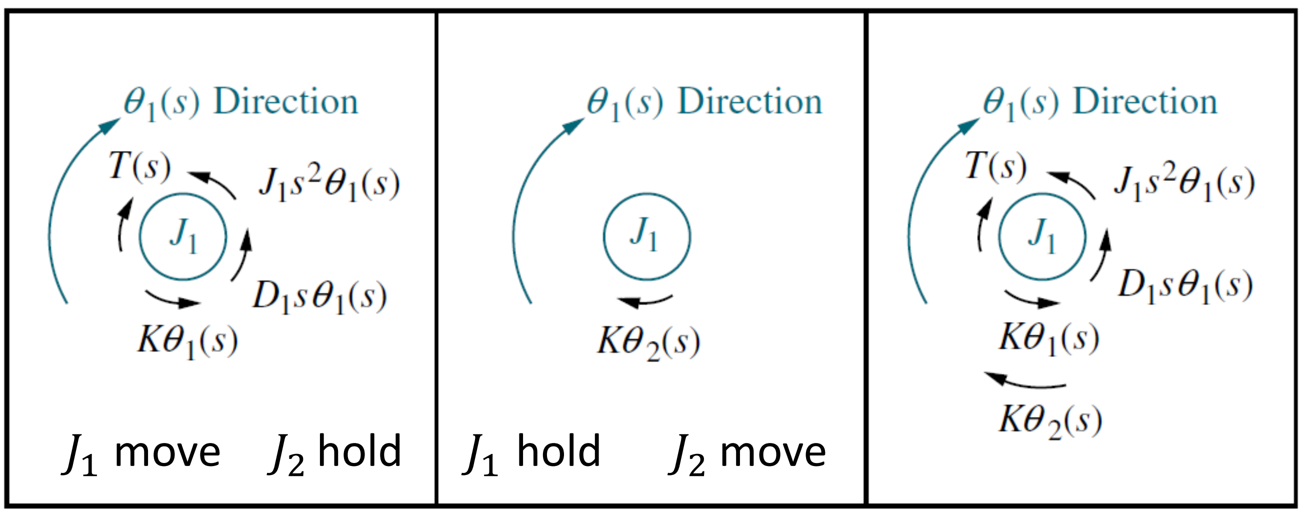
Mechanical system (5)

Example Find $G(s) = \theta_2(s)/T(s)$



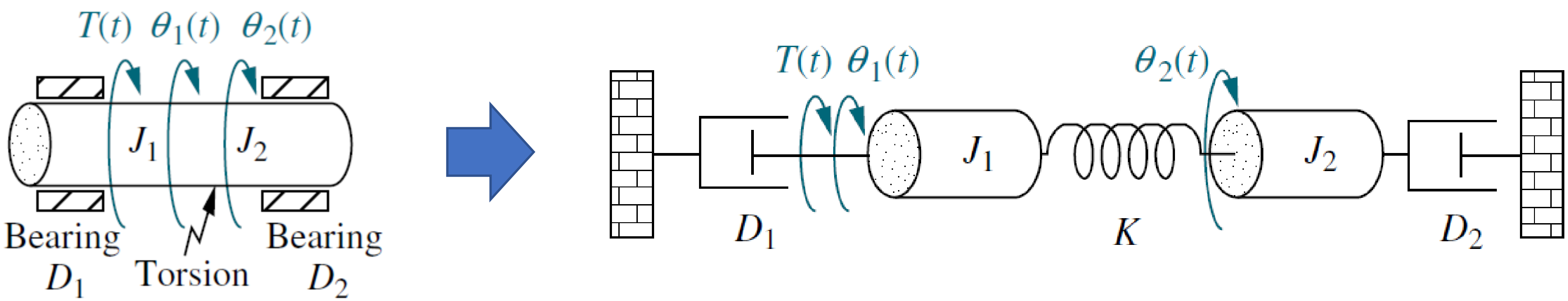


J_1

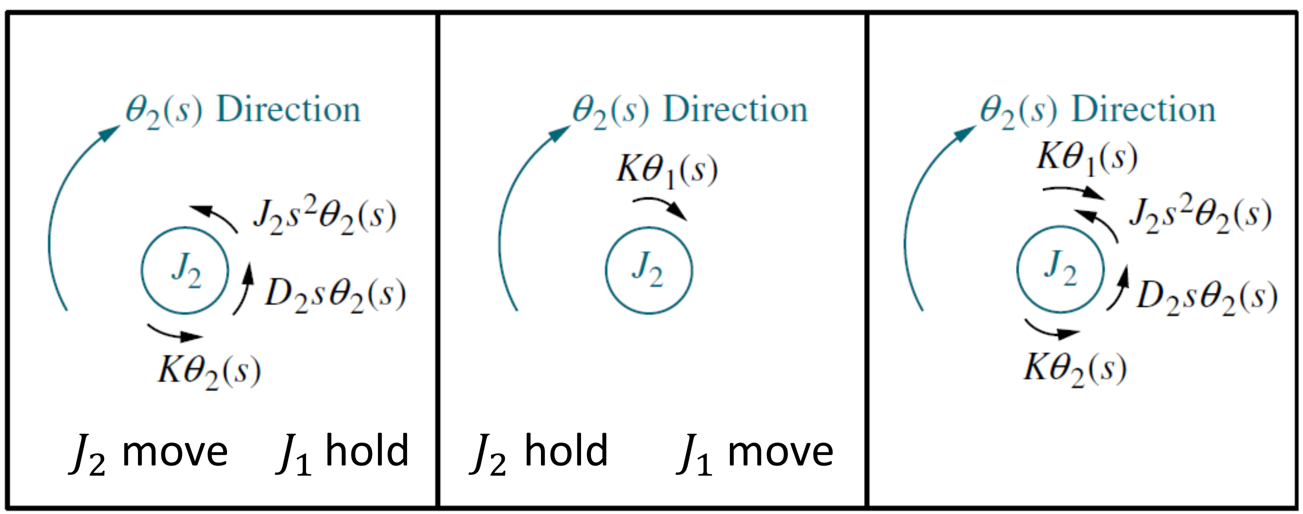


Mechanical system (6)

Example Find $G(s) = \theta_2(s)/T(s)$

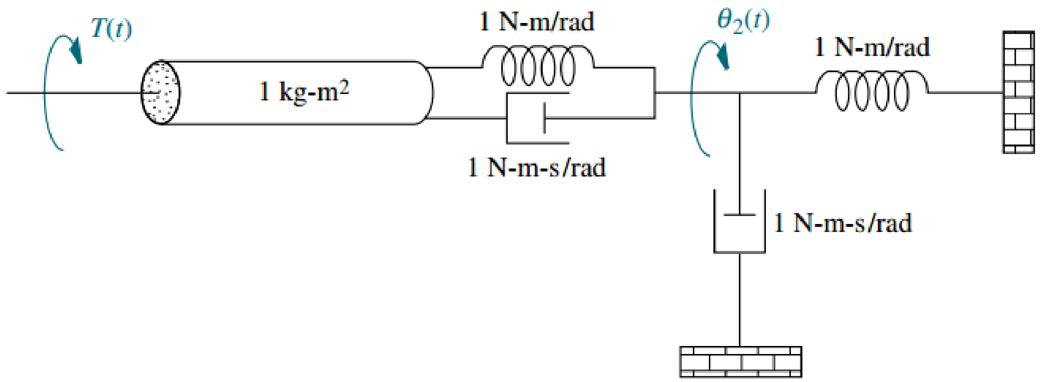


$\boldsymbol{J_2}$



Mechanical system (7)

Find $G(s) = \theta_2(s)/T(s)$



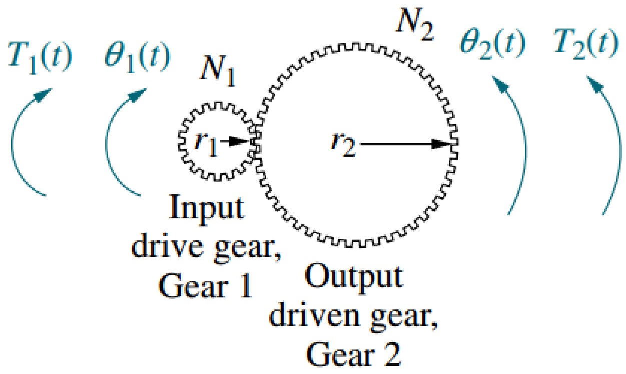
- J_1 move J_2 hold
- J_1 hold J_2 move
- J_2 move J_1 hold
- J_2 hold J_1 move

Mechanical system (8)

- **Speed**

$$r_1\theta_1 = r_2\theta_2$$

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

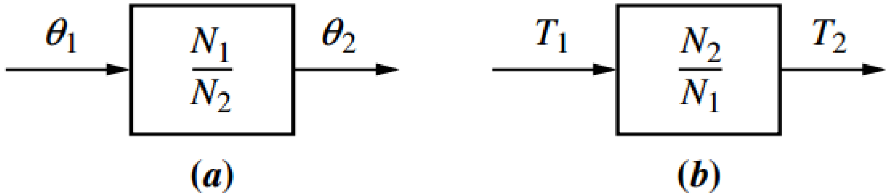


- **Work**

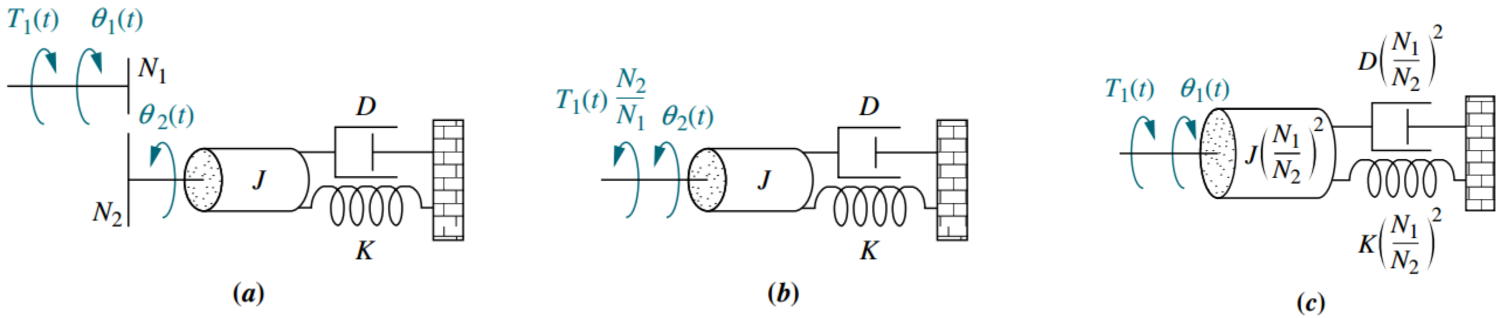
$$T_1\theta_1 = T_2\theta_2$$

cf.) W= Fx

$$\frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$$



Mechanical system (9)



$$(Js^2 + Ds + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

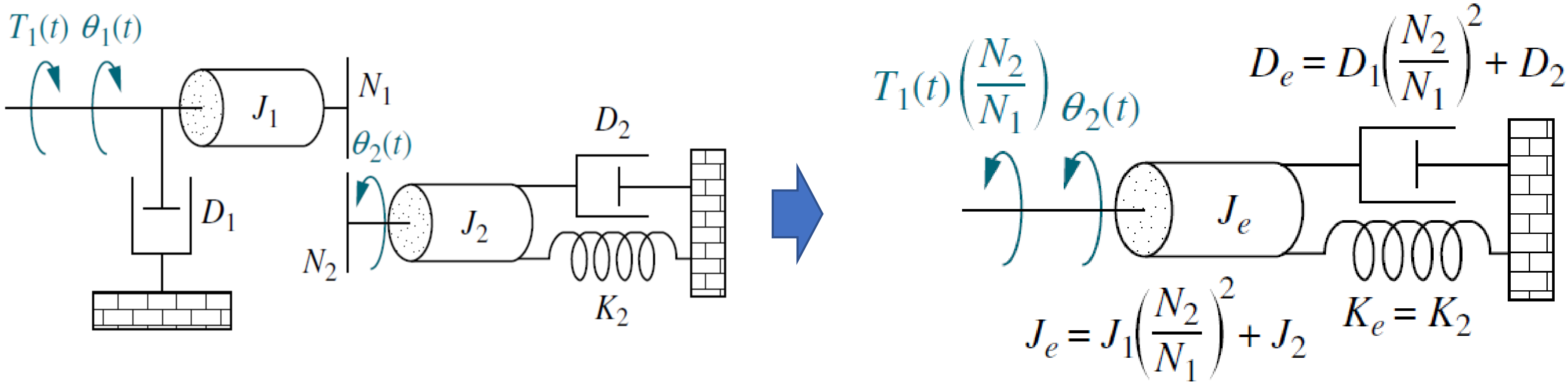
$$(Js^2 + Ds + K)\frac{N_1}{N_2}\theta_1(s) = T_1(s)\frac{N_2}{N_1}$$

$$\left[J\left(\frac{N_1}{N_2}\right)^2 s^2 + D\left(\frac{N_1}{N_2}\right)^2 s + K\left(\frac{N_1}{N_2}\right)^2\right]\theta_1(s) = T_1(s)$$

$$\left(\frac{\text{Number of teeth of gear on } \textit{destination} \text{ shaft}}{\text{Number of teeth of gear on } \textit{source} \text{ shaft}}\right)^2$$

Mechanical system (10)

Find $G(s) = \theta_2(s)/T_1(s)$



$$(J_e s^2 + D_e s + K_e)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

$$J_e = J_1\left(\frac{N_2}{N_1}\right)^2 + J_2; \quad D_e = D_1\left(\frac{N_2}{N_1}\right)^2 + D_2; \quad K_e = K_2$$

$$G(s) = \frac{\theta_2(s)}{T_1(s)} = \frac{N_2/N_1}{J_e s^2 + D_e s + K_e}$$

Electrical vs Mechanical

Same principle !!

$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } \theta_1\end{bmatrix}\theta_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ \theta_1 \text{ and } \theta_2\end{bmatrix}\theta_2(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied torques}\\ \text{at } \theta_1\end{bmatrix} \quad (2.129a)$$

$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ \theta_1 \text{ and } \theta_2\end{bmatrix}\theta_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } \theta_2\end{bmatrix}\theta_2(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied torques}\\ \text{at } \theta_2\end{bmatrix} \quad (2.129b)$$

$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_1\end{bmatrix}X_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_2\end{bmatrix}X_2(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_1\end{bmatrix} \quad (2.120a)$$

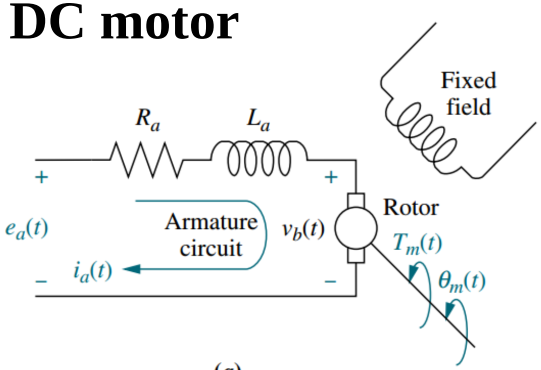
$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_2\end{bmatrix}X_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_2\end{bmatrix}X_2(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_2\end{bmatrix} \quad (2.120b)$$

$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 1}\end{bmatrix}I_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to the}\\ \text{two meshes}\end{bmatrix}I_2(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{voltages around}\\ \text{Mesh 1}\end{bmatrix} \quad (2.83a)$$

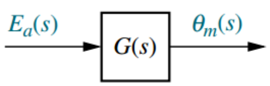
$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to the}\\ \text{two meshes}\end{bmatrix}I_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 2}\end{bmatrix}I_2(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{voltages around}\\ \text{Mesh 2}\end{bmatrix} \quad (2.83b)$$

Electromechanical system (1)

- **DC motor**



(a)



(b)

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

Electrical

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s)$$

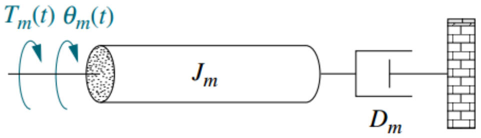
$$v_b(t) = K_b \frac{d\theta_m(t)}{dt}$$

$$T_m(s) = K_t I_a(s)$$

$$\frac{(R_a + L_a s)T_m(s)}{K_t} + K_b s \theta_m(s) = E_a(s)$$

Mechanical

$$T_m(s) = (J_m s^2 + D_m s)\theta_m(s)$$

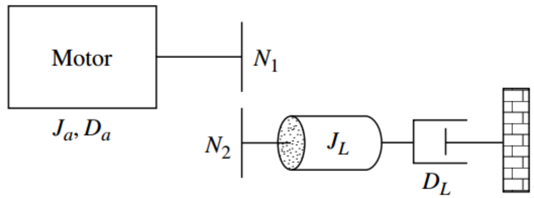


Electromechanical system (2)

- **How to evaluate constants of motor?**

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]} \Rightarrow \left(J_m,\ D_m,\ \frac{K_t}{R_a},\ K_b\right)$$

- Mechanical



$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2; \quad D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2$$

- Electrical

$$L_a = 0, \quad \frac{R_a}{K_t}T_m(s) + K_b s\theta_m(s) = E_a(s)$$

$$T_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a}e_a$$

$$T_{\text{stall}} = \frac{K_t}{R_a}e_a$$

$$\omega_{\text{no-load}} = \frac{e_a}{K_b}$$

$$\frac{K_t}{R_a} = \frac{T_{\text{stall}}}{e_a}$$

$$K_b = \frac{e_a}{\omega_{\text{no-load}}}$$