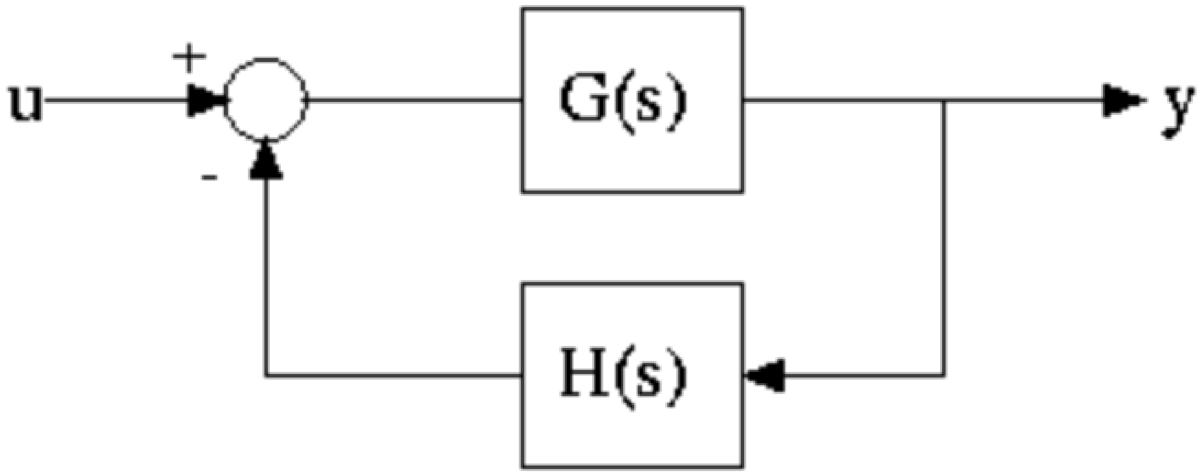
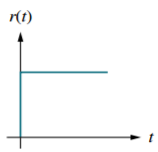
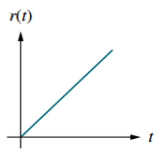
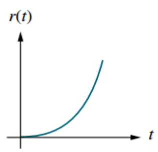
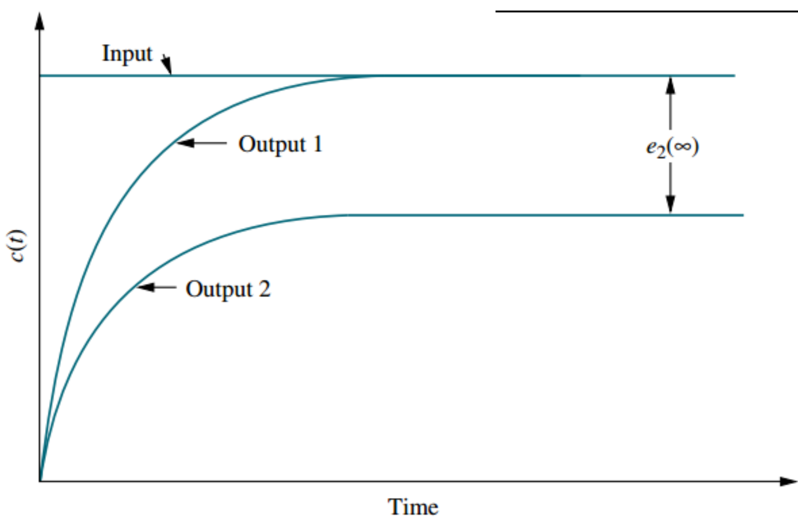
System Control

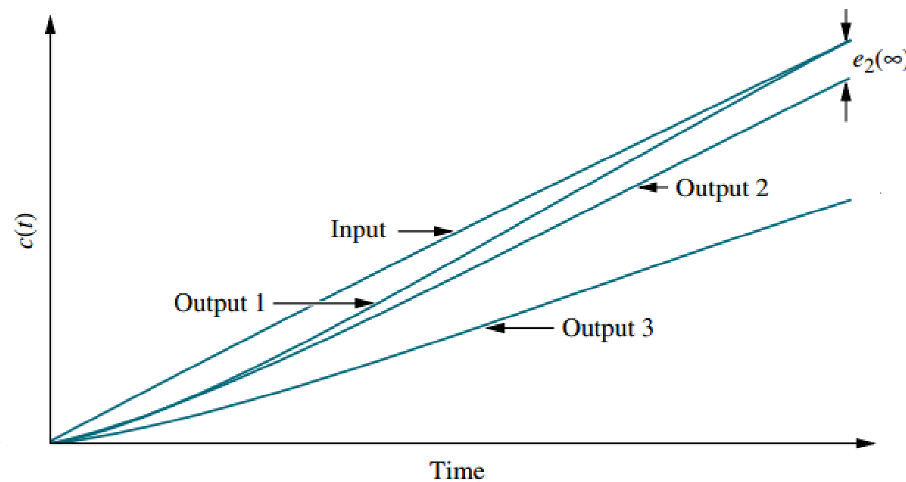
Steady-state errors



Steady-state error

Waveform	Name	Physical interpretation	Time function	Laplace transform
	Step	Constant position	1	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$





Representation

Final value theorem

$$\mathscr{L}[\dot{f}(t)] = \int_{0-}^{\infty} \dot{f}(t)e^{st}dt = sF(s) - f(0-)$$

As $s \rightarrow 0$,

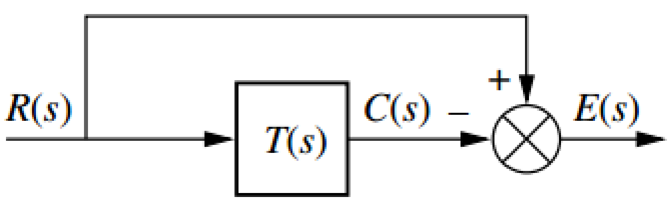
$$\int_{0-}^{\infty} \dot{f}(t)dt = f(\infty) - f(0-) = \lim_{s\rightarrow 0} sF(s) - f(0-) \quad \text{or} \quad f(\infty) = \lim_{s\rightarrow 0} sF(s)$$

- In terms of T(s)

$$E(s) = R(s) - C(s)$$

$$C(s) = R(s)T(s)$$

$$e(\infty) = \lim_{s\rightarrow\infty} sR(s)[1 - T(s)]$$



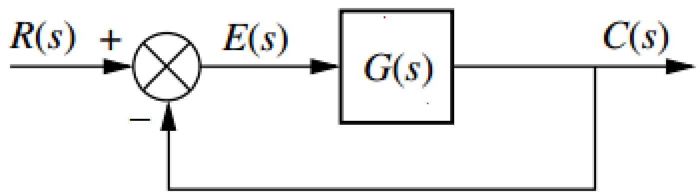
- In terms of G(s)

$$E(s) = R(s) - C(s)$$

$$C(s) = E(s)G(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)}$$

$$e(\infty) = \lim_{s\rightarrow 0} \frac{sR(s)}{1 + G(s)}$$



Steady-state error in terms of G(s)

Step Input. with $R(s) = 1/s$, we find

$$e(\infty) = e_{\text{step}}(\infty) = \lim_{s\to 0} \frac{s(1/s)}{1+G(s)} = \frac{1}{1+\lim_{s\to 0} G(s)}$$

$$\lim_{s\to 0} G(s) = \infty$$

Ramp Input. with, $R(s) = 1/s^2$, we obtain

$$e(\infty) = e_{\text{ramp}}(\infty) = \lim_{s\to 0} \frac{s(1/s^2)}{1+G(s)} = \lim_{s\to 0} \frac{1}{s+sG(s)} = \frac{1}{\lim_{s\to 0} sG(s)}$$

$$\lim_{s\to 0} sG(s) = \infty$$

Parabolic Input. with $R(s) = 1/s^3$, we obtain

$$e(\infty) = e_{\text{parabola}}(\infty) = \lim_{s\to 0} \frac{s(1/s^3)}{1+G(s)} = \lim_{s\to 0} \frac{1}{s^2+s^2G(s)} = \frac{1}{\lim_{s\to 0} s^2G(s)}$$

$$\lim_{s\to 0} s^2G(s) = 0$$

Static error constants and system type

Static Error Constants

Given unity feedback system, we have the following relationships:

For a step input $R(t) = u(t)$, $e(\infty) = e_{step}(\infty) = \dfrac{1}{1 + \lim_{s \to 0} G(s)}$

- Position constant $K_p = \lim_{s \to 0} G(s)$

For a ramp input $R(t) = tu(t)$, $e(\infty) = e_{ramp}(\infty) = \dfrac{1}{\lim_{s \to 0} sG(s)}$

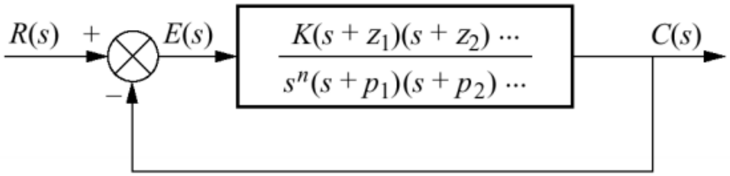
- Velocity constant $K_v = \lim_{s \to 0} sG(s)$

For a parabolic input $R(t) = \dfrac{1}{2} t^2 u(t)$, $e(\infty) = e_{parabola}(\infty) = \dfrac{1}{\lim_{s \to 0} s^2 G(s)}$

- Acceleration constant $K_a = \lim_{s \to 0} s^2 G(s)$

System Type

We define *system type* to be the value of n in the denominator

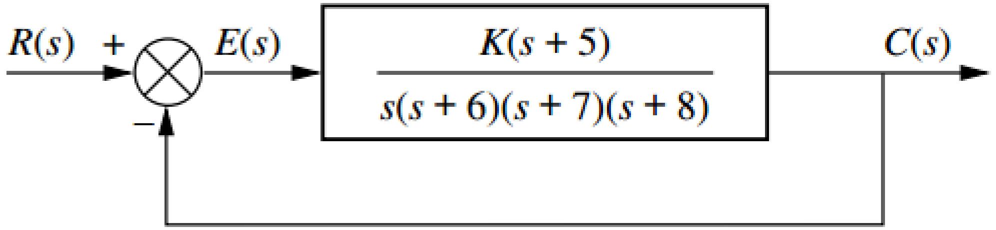


Static error constants and system type

Input	Steady-state error formula	Type 0 Static error constant	Type 0 Error	Type 1 Static error constant	Type 1 Error	Type 2 Static error constant	Type 2 Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p = \text{Constant}$	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v = \text{Constant}$	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = \text{Constant}$	$\frac{1}{K_a}$

Gain Design to Meet a Steady-State Error Specification

PROBLEM: Given the control system in Figure, find the value of K so that there is 10% error in the steady state.



Review : Modeling

- System

  Electrical system: Ohm's Law, Kirchhoff's Law

  Mechanical system: Newton's Law

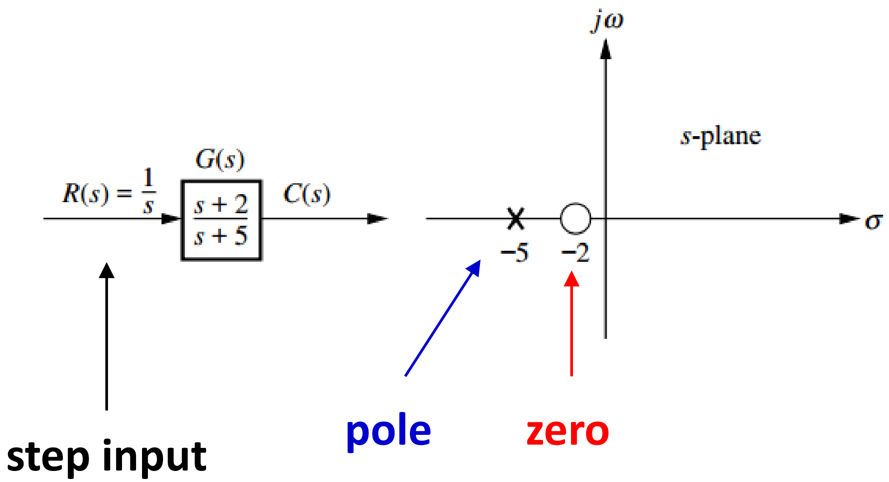
  Electrical system + Mechanical system

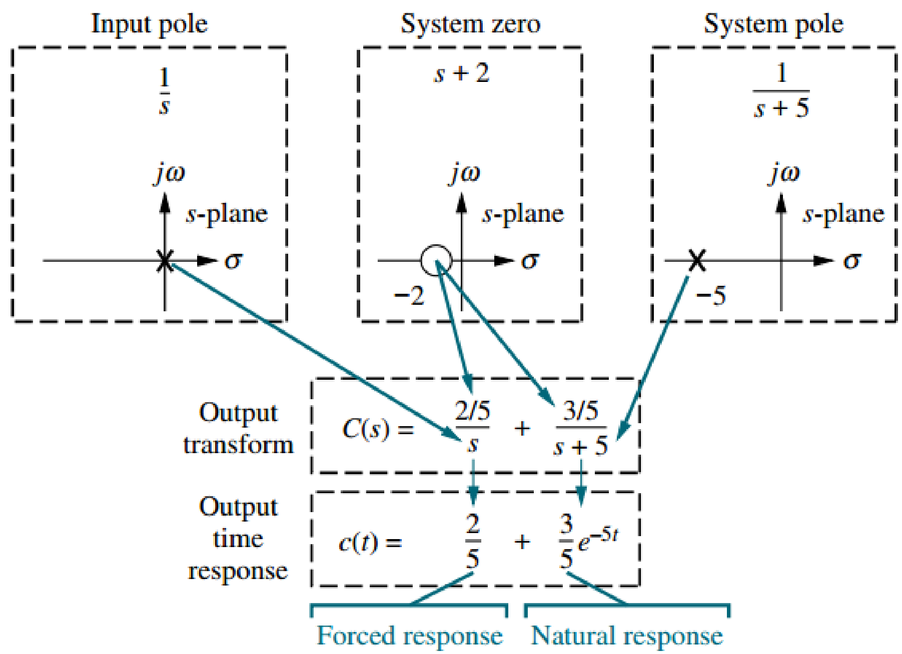
  = Electromechanical system (ex: Motor)

- System modeling

  Modeling in the [frequency domain / time domain

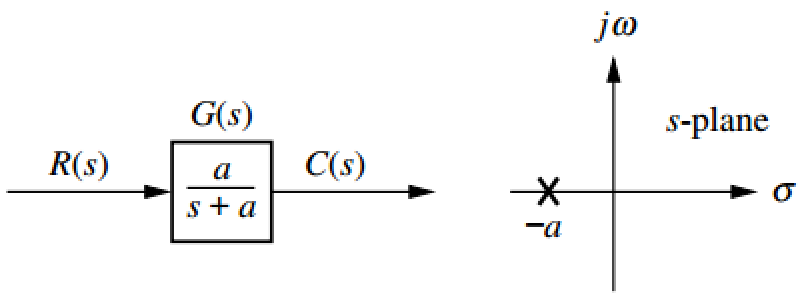
Review : Poles, zeros, and system response





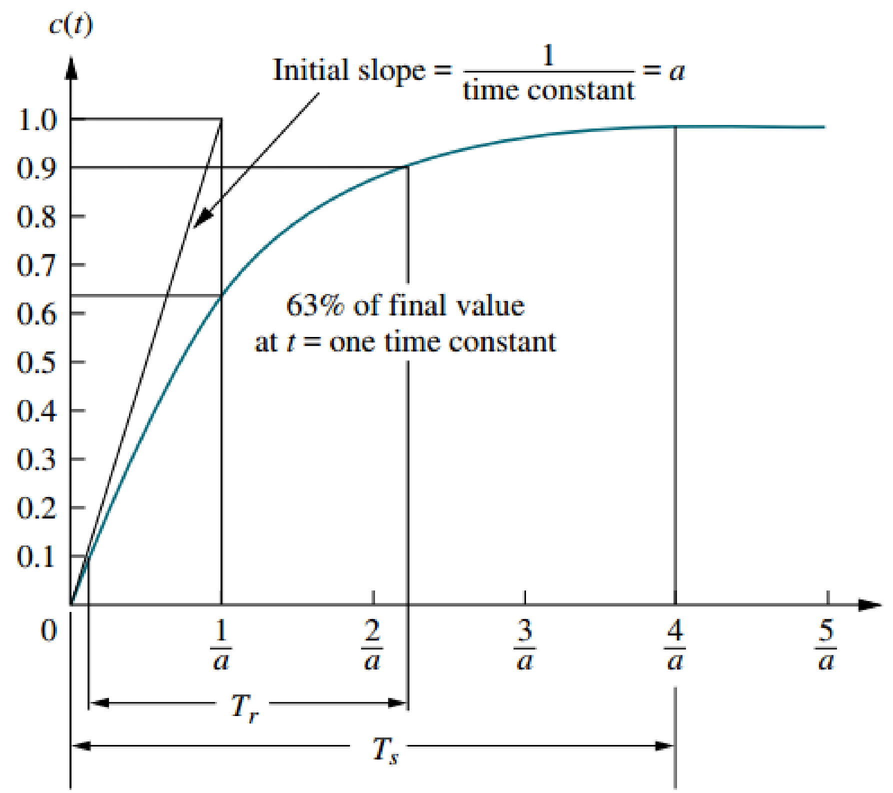
- Input poles: determine the form of the steady state response
- System poles: determines the form of the transient response

Review : First-order system



$$C(s) = R(s)G(s) = \frac{a}{s(s+a)}$$

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

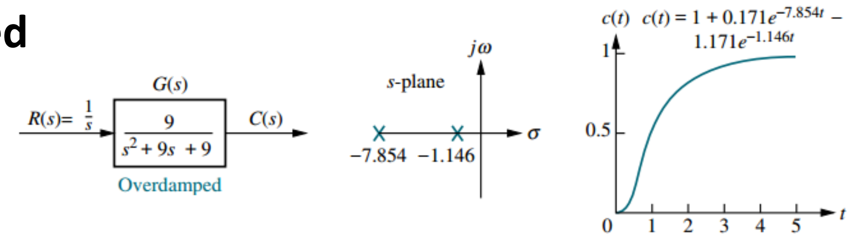


1. Time constant
2. Rise time
3. Settling time

Review : Second-order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

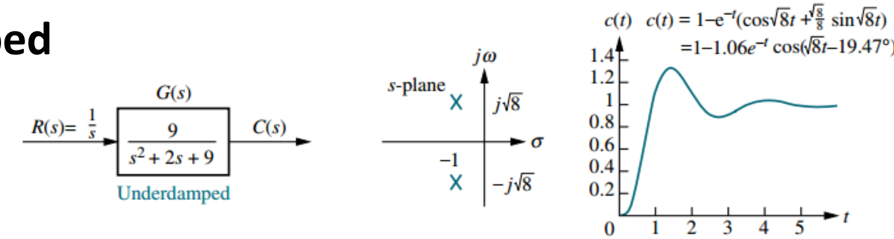
Overdamped



Poles: Two real at $-\sigma_1, \ -\sigma_2$

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

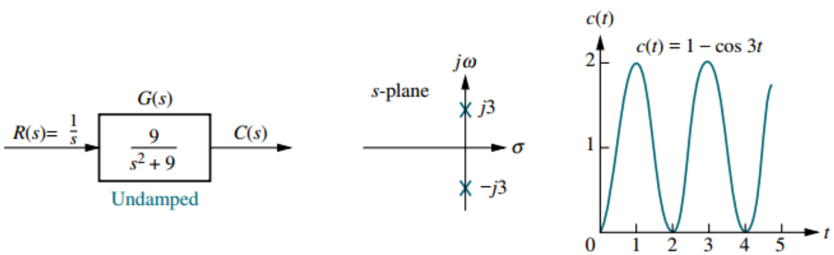
Underdamped



Poles: Two complex at $-\sigma_d \pm j\omega_d$

$$c(t) = Ae^{-\sigma_d t}\cos(\omega_d t - \phi)$$

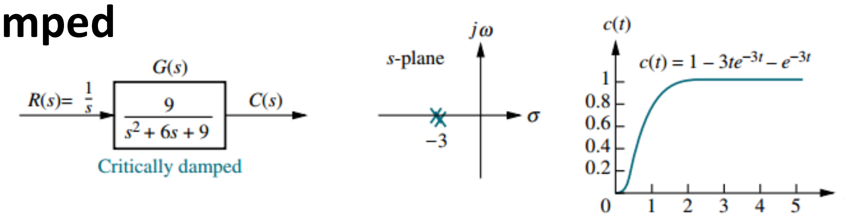
Undamped



Poles: Two imaginary at $\pm j\omega_1$

$$c(t) = A\cos(\omega_1 t - \phi)$$

Critically damped

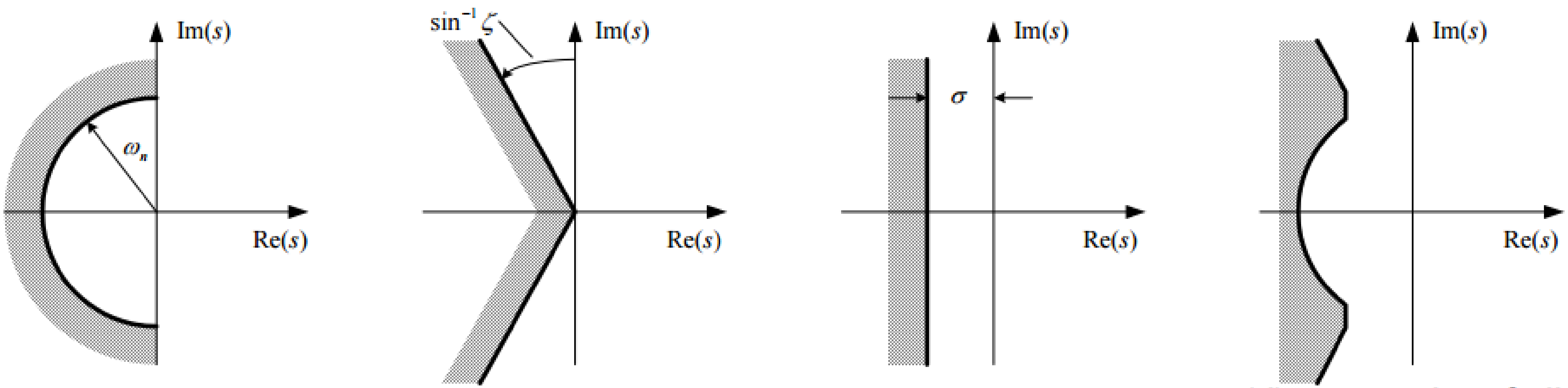


Poles: Two real at $-\sigma_1$

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 t e^{-\sigma_1 t}$$

Review : Time domain design specification

1. Peak time: $$T_p = \frac{\pi}{\omega_d}$$
2. Percent overshoot: $$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$
3. Rise time: $$\omega_n T_r = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1$$
4. Settling time: $$T_s = \frac{4}{\sigma_d}$$



(a) rise time (b) overshoot (c) settling time (d) composite of all three requirements